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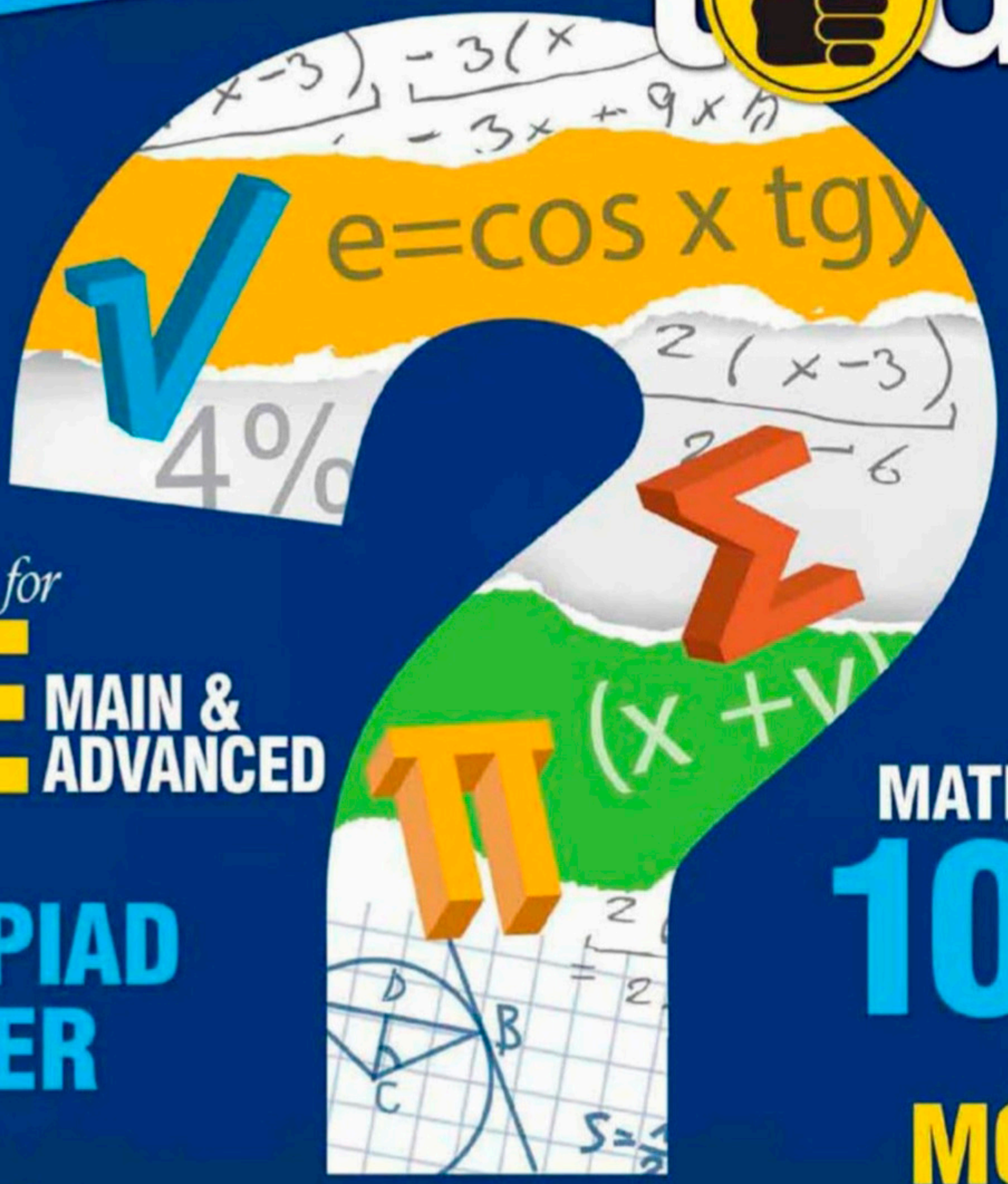

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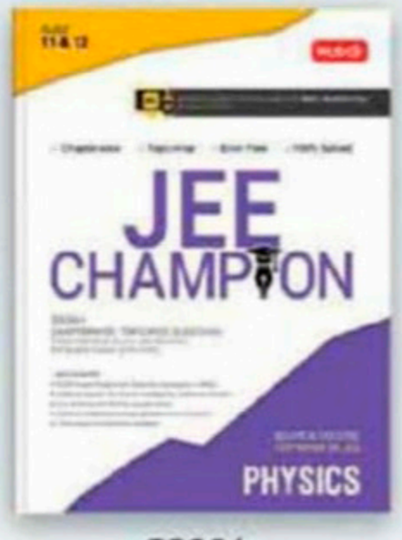

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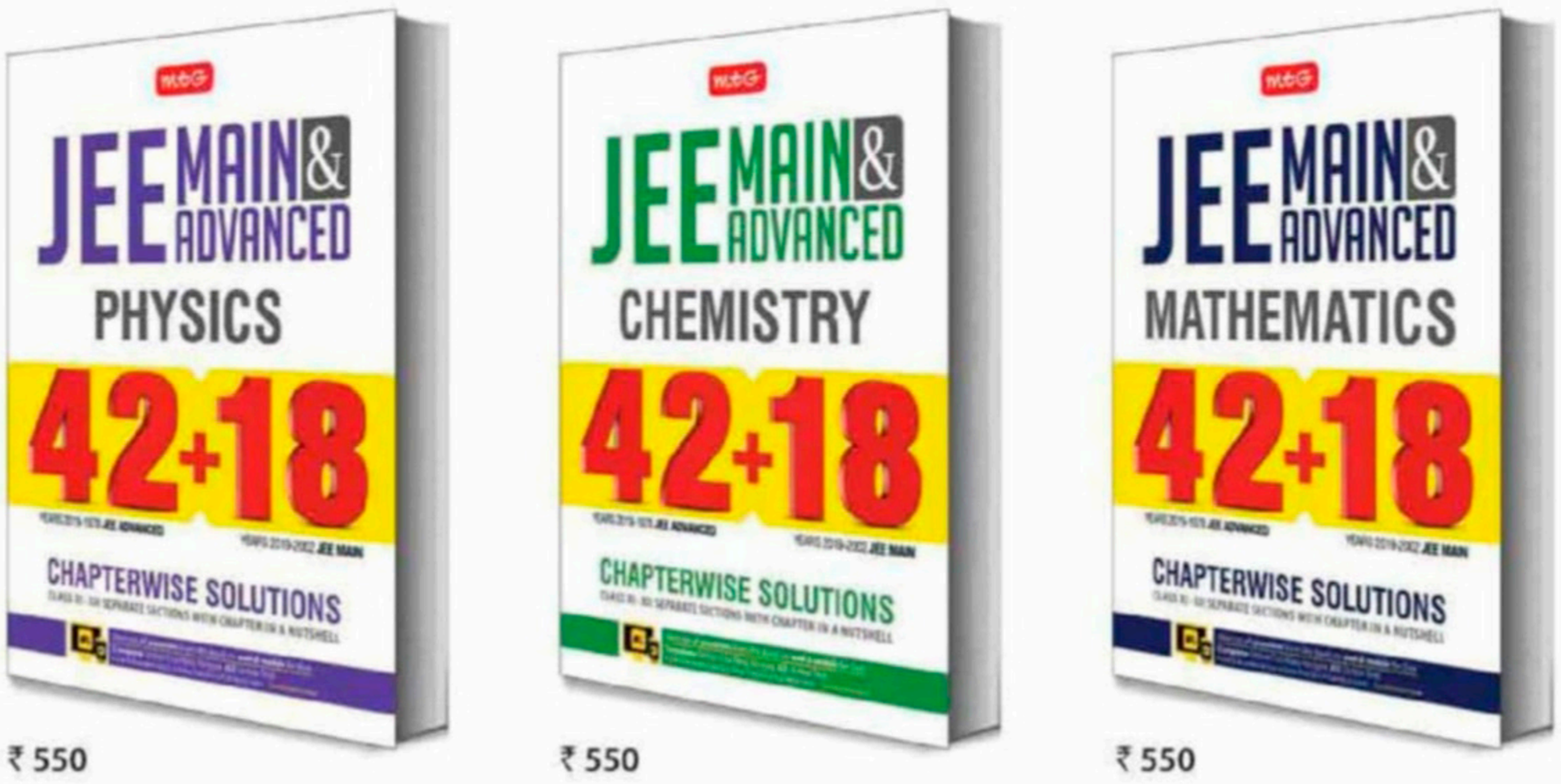

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CONTENTS

Competition Edge

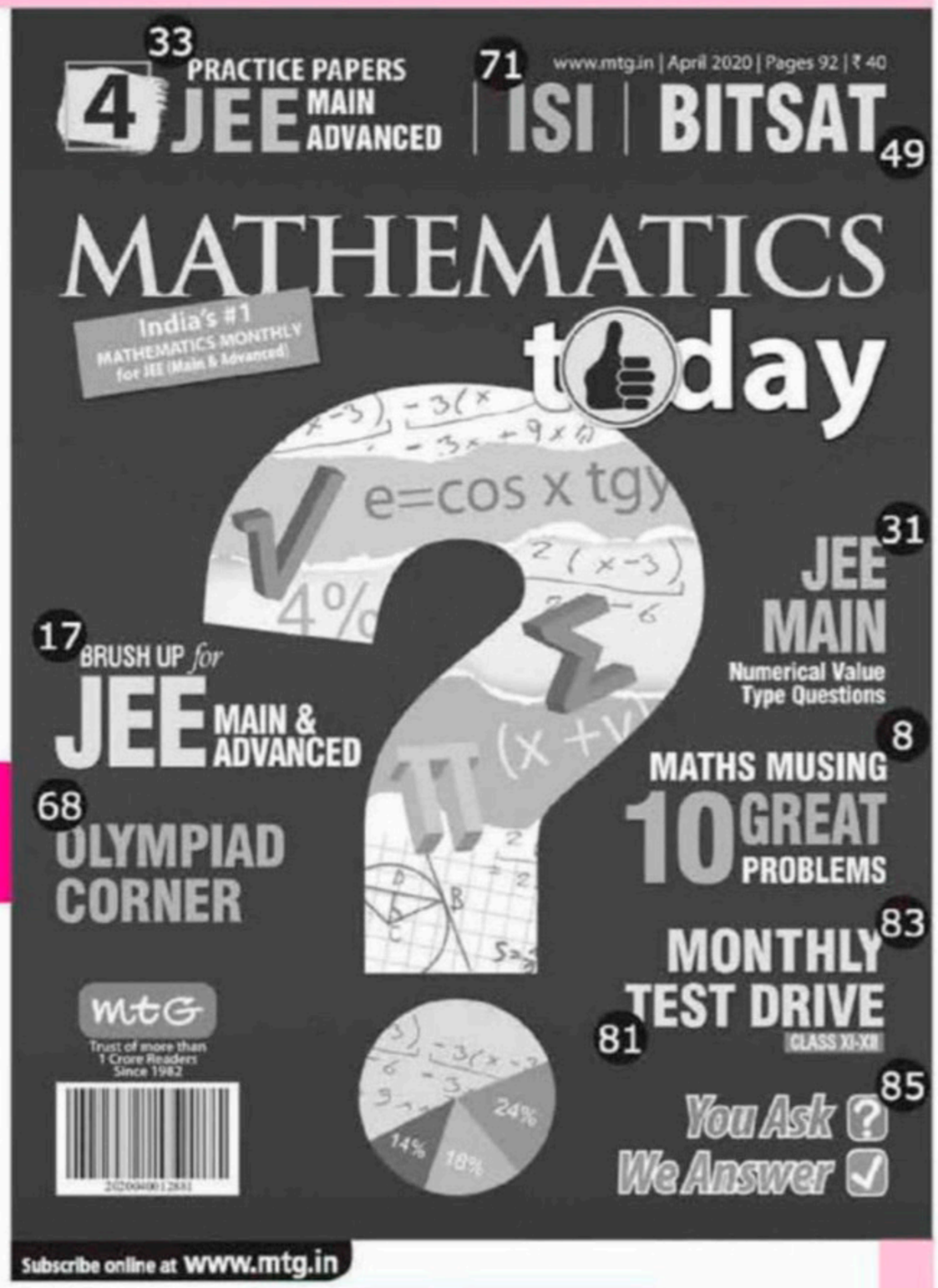
8 Maths Musing Problem Set - 208
10 Mock Test Paper JEE Main 2020
17 Brush Up for JEE 2020 (Unit 10)
30 Maths Musing Solutions
31 JEE Main 2020
Numerical Value Type Questions
33 JEE Advanced Practice Paper 2020
39 Practice Problems JEE Advanced 2020
49 BITSAT Practice Paper 2020
65 Challenging Problems
68 Olympiad Corner
71 Mock Test Paper ISI
85 You Ask We Answer

Class XI

46 Concept Map
81 Monthly Test Drive

Class XII

47 Concept Map
83 Monthly Test Drive



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During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 208

JEE MAIN

1. $(1 - \cot 22°)(1 - \cot 23°) =$

(a) 1 (b) 2

(c) 3 (d) 4

2. Angle between $x = 2$ and $x - 3y = 6$ is

(a) ∞ (b) $\tan^{-1}(3)$

(c) $\tan^{-1}(1/3)$ (d) none of these

3. $\int \frac{dx}{x \log x \cdot \log(\log x)} =$

(a) $2\log(\log x) + c$ (b) $\log[\log(\log x)] + c$

(c) $\log(x \log x) + c$ (d) none of these

4. The solution of the differential equation $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$ is

(a) $\tan y = c(1 - e^x)^3$ (b) $(1 - e^x)^3 \tan y = c$

(c) $\tan y = c(1 - e^x)$ (d) $(1 - e^x)\tan y = c$

5. If the roots of $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then a, b, c are in

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

JEE ADVANCED

6. If $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$, then the value of $a_0 + a_3 + a_6 + ...$, is

(a) $a_1 + a_4 + a_7 + ...$

(b) $a_2 + a_5 + a_8 + ...$

(c) 3^{n-1}

(d) none of these

COMPREHENSION

Let A, B, C be three sets of complex numbers as defined below:

$A = \{z : \text{Im } z \geq 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$

7. The number of elements in the set $A \cap B \cap C$ is

(a) 0 (b) 1

(c) 2 (d) ∞

8. Let z be any point in $A \cap B \cap C$. Then $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(a) 25 and 29 (b) 30 and 34

(c) 35 and 39 (d) 40 and 44

NUMERICAL VALUE TYPE

9. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is such that the length of tangents to it from the points $(1, 0)$, $(2, 0)$ and $(3, 2)$ are respectively $1, \sqrt{7}$ and $\sqrt{2}$. Then the value of $|4f|$ is _____.

MATRIX MATCH

10. Consider the functions $f_1(x) = \sin^{-1}(\sin x)$, $f_2(x) = \sin^{-1}(\cos x)$, $f_3(x) = \cos^{-1}(\sin x)$, $f_4(x) = \cos^{-1}(\cos x)$. Column-I gives the values of x and Column-II gives the values of derivatives $f_1'(x), f_2'(x), f_3'(x), f_4'(x)$ in that order.

Match the following columns :

Column-I		Column-II	
P.	3	1.	$-1, -1, 1, 1$
Q.	4	2.	$-1, 1, 1, -1$
R.	5	3.	$1, -1, -1, 1$
S.	7	4.	$1, 1, -1, -1$

	P	Q	R	S
(a)	1	3	4	2
(b)	2	1	3	4
(c)	1	2	4	3
(d)	1	2	3	4

See Solution Set of Maths Musing 207 on page no. 30

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MOCK TEST PAPER 2020

JEE MAIN

With Numerical Value Type Questions

Time : 1 hr. 15 min.

1. A man has seven relatives, 4 of them are ladies and 3 are gentlemen, his wife has 7 relatives out of which 3 are ladies and 4 are gentlemen. The number of ways in which they can invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of the man's relative and 3 of the wife's relative, are

(a) 144 (b) 340 (c) 484 (d) 485

2. In a group of 45 persons, 25 drink tea but not coffee, while 32 drink tea. How many persons drink coffee but not tea?

(a) 12 (b) 13 (c) 15 (d) 20

3. If $\int x\log\left(1+\frac{1}{x}\right)dx = f(x)\cdot\log(x+1) + g(x)\log x + Lx + C$, then

(a) $f(x)=\frac{x^2}{2}$ (b) $g(x) = \log_e x$

(c) $L = 1$ (d) $L=\frac{1}{2}$

4. Let $A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$. If $|A^2|=25$, then $|\alpha|=$

(a) $\frac{1}{5}$ (b) 5 (c) 5^2 (d) 1

5. If $\frac{1+3p}{4}, \frac{1-p}{3}, \frac{1-3p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is

(a) $\left[-\frac{1}{3},\frac{1}{3}\right]$ (b) $\left[-\frac{1}{3},1\right]$

(c) $\left[\frac{1}{13},1\right]$ (d) $\left[\frac{1}{13},\frac{1}{3}\right]$

6. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $(\vec{a}\times\vec{b})^2 =$

(a) 48 (b) 8

(c) 16 (d) none of these

7. If α and β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are α^2 and β^2 is

(a) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(b) $a^2x^2 + (b^2 - 2ac)x + c^2 = 0$

(c) $a^2x^2 + (b^2 + ac)x + c^2 = 0$

(d) $a^2x^2 + (b^2 + 2ac)x + c^2 = 0$

8. If $f(x)=\begin{cases}\dfrac{\sin\{\cos x\}}{x-\frac{\pi}{2}}, & x\neq\dfrac{\pi}{2}\\ 1, & x=\dfrac{\pi}{2}\end{cases}$, where $\{\cdot\}$ represents the fractional part function, then $f(x)$ is

(a) continuous at $x=\frac{\pi}{2}$

(b) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x=\frac{\pi}{2}$

(c) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ does not exist

(d) $\lim\limits_{x\to\frac{\pi}{2}^+} f(x) = -1$

9. For a differentiable function f,

$\lim\limits_{h\to0}\frac{\{f(x+h)\}^2 - \{f(x)\}^2}{2h} =$

(a) $\{f'(x)\}^2$ (b) $\frac{1}{2}\{f'(x)\}^2$

(c) $\frac{1}{2}[\{f'(x)\}^2 - \{f(x)\}^2]$

(d) $f(x)f'(x)$

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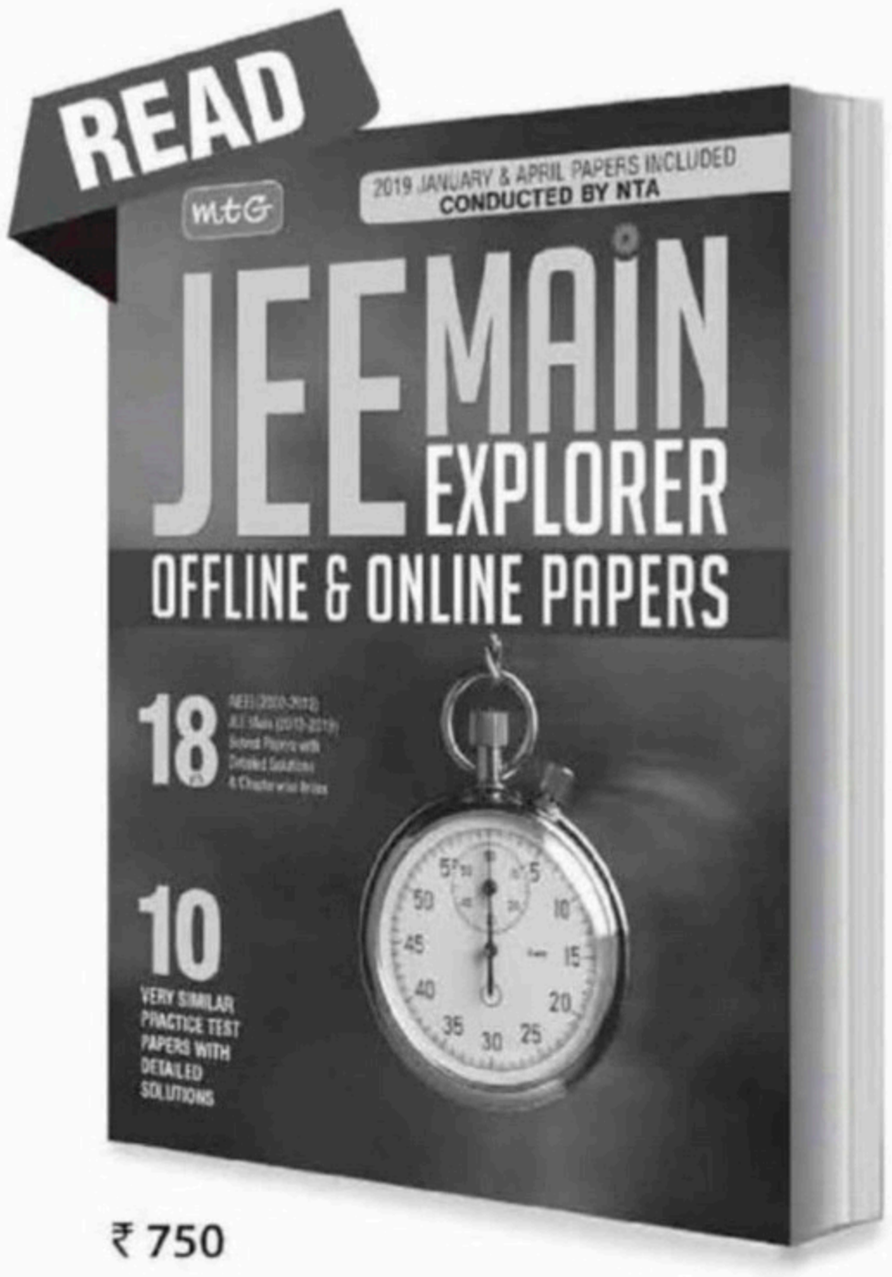

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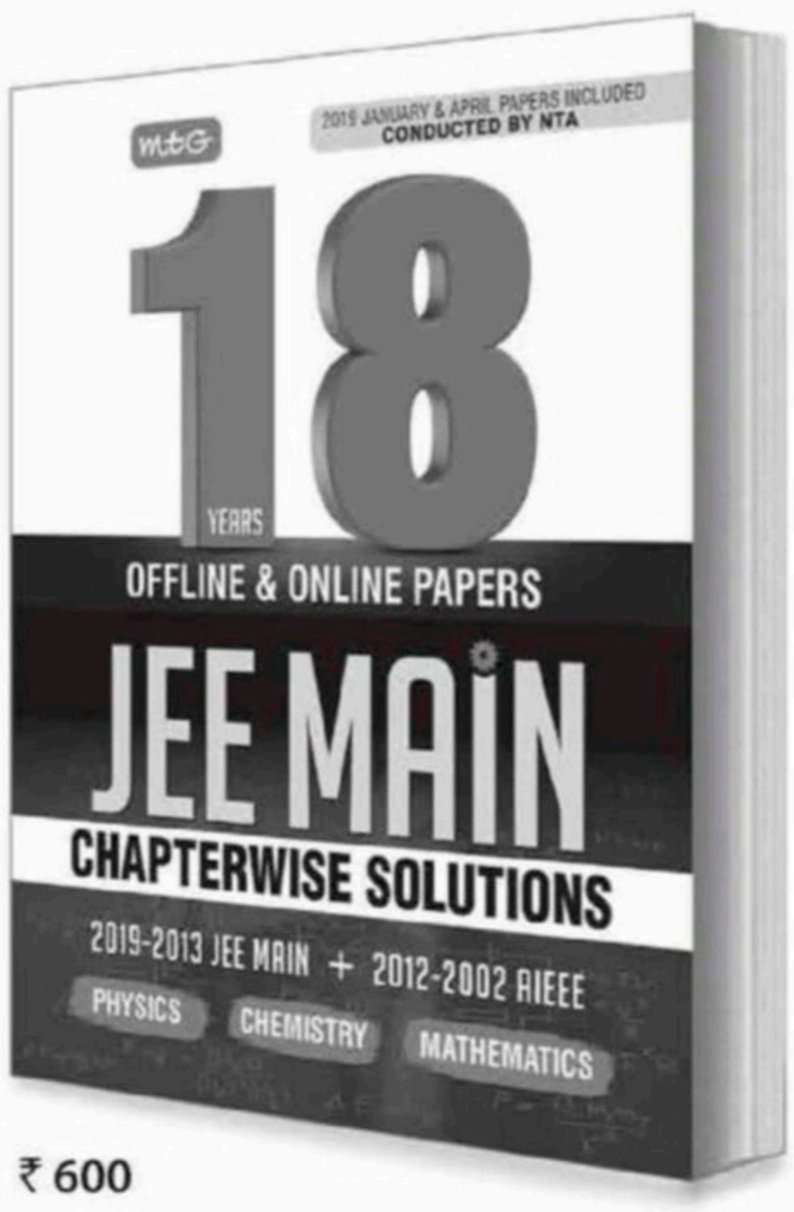

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10. The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is

(a) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{\lfloor n}\cdot x^n$ (b) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n+1)}{\lfloor n}\cdot 2^n$

(c) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{\lfloor(n+1)}\cdot 2^n$ (d) $\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{\lfloor n}\cdot 2^n$

11. PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola, then the eccentricity (e) of the hyperbola satisfies

(a) $1<e<\frac{2}{\sqrt{e}}$ (b) $e=\frac{2}{\sqrt{3}}$

(c) $e=\frac{\sqrt{3}}{2}$ (d) $e>\frac{2}{\sqrt{3}}$

12. An ellipse has eccentricity $\frac{1}{2}$ and one focus at $S\left(\frac{1}{2},1\right)$. Its one directrix is the common tangent, (nearer to S) to the circle $x^2+y^2=1$ and rectangular hyperbola $x^2-y^2=1$. The equation of the ellipse in standard form is

(a) $9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$

(b) $12\left(x-\frac{1}{3}\right)^2+9(y-1)^2=1$

(c) $\frac{\left(x-\frac{1}{2}\right)^2}{12}+\frac{(y-1)^2}{9}=1$

(d) $3\left(x+\frac{1}{2}\right)^2+4(y-1)^2=1$

13. $f(x)=\begin{cases}3[x]-5\frac{|x|}{x}, & x\neq0\\ 2, & x=0\end{cases}$, then $\int_{-3/2}^{2} f(x)dx$ is equal to (where, $[\cdot]$ denotes the greatest integer function)

(a) $-\frac{11}{2}$ (b) $-\frac{7}{2}$ (c) -6 (d) $-\frac{17}{2}$

14. Let the population of rabbits surviving at a time be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$. If $p(0)=100$, then $p(t)=$

(a) $400-300e^{\frac{t}{2}}$ (b) $300-200e^{-\frac{t}{2}}$

(c) $600-500e^{\frac{t}{2}}$ (d) $400-300e^{-\frac{t}{2}}$

15. A sample of 4 items is drawn at random from a lot of 10 items, containing 3 defectives. If x denotes the number of defective items in the sample, then $p(0<x<3)$ is equal to

(a) 3/10 (b) 1/2 (c) 4/5 (d) 1/6

16. The principal argument of the complex number $z=\frac{1+\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}}{1+\sin\frac{\pi}{3}-i\cos\frac{\pi}{3}}$ is

(a) $\pi/3$ (b) $\pi/6$ (c) $2\pi/3$ (d) $\pi/2$

17. The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

18. For any two real numbers a and b, we define aRb if and only if $\sin^2a+\cos^2b=1$. The relation R is

(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

19. A mapping from N to N is defined as follows $f:N\to N, f(n)=(n+5)^2, n\in N$ (N is the set of natural numbers). Then

(a) f is not one to one
(b) f is onto
(c) f is both one to one and onto
(d) f is one to one but not onto

20. The distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$ is

(a) 1 (b) 2
(c) 4 (d) none of these

NUMERICAL VALUE TYPE

21. The value of $-\cos\left[2\tan^{-1}\frac{1+x}{1-x}+\sin^{-1}\frac{1-x^2}{1+x^2}\right]$ is _____.

22. $x+8y-22=0, 5x+2y-34=0$ and $2x-3y+13=0$ are the three sides of a triangle. The area of the triangle is _____.

23. The four common tangents to the ellipses $\frac{x^2}{9}+\frac{y^2}{4}=1$ and $\frac{x^2}{4}+\frac{y^2}{9}=1$ form a square of area _____.

24. Let $f(x) = ax^3 + bx^2 + cx$ has relative extrema at $x = 1$ and $x = 5$. If $\int_{-1}^{1} f(x)\,dx = 6$, then $a+b =$ _____.

25. For a fixed value of $n = 20$, the maximum value of variance of binomial distribution is _____.

SOLUTIONS

1. (d) : The number of ways of selections of 3 man's relative and 3 wife's relative are shown in the following table :

Cases	Man's relative(7)		Wife's relative(7)	
	3 gentlemen	4 ladies	4 gentlemen	3 ladies
(i)	0	3	3	0
(ii)	1	2	2	1
(iii)	2	1	1	2
(iv)	3	0	0	3

The required number of ways

$= {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0 + {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1$
$+ {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 + {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3$
$= 16 + 324 + 144 + 1 = 485$

2. (b) : Let T and C denote the persons who drink tea and coffee respectively.

Given, $n(T \cup C) = 45$ ...(i)

$n(T \cap C') = 25$ ...(ii)

$n(T) = 32$ ...(iii)

From (ii), we have $n(T \cap C') = 25$

$\Rightarrow n(T) - n(T \cap C) = 25$

$\therefore n(T \cap C) = 32 - 25$ [Using (iii)]

$\Rightarrow n(T \cap C) = 7$ ...(iv)

Again from (i), we have

$n(T \cup C) = n(T) + n(C) - n(T \cap C) = 45$

$\Rightarrow 32 + n(C) - 7 = 45$ [Using (iii) and (iv)]

$\therefore n(C) = 20$

$\therefore n(C \cap T') = n(C - T) = n(C) - n(C \cap T)$
$= 20 - 7 = 13$

3. (d) : Let $I = \int x \log\left(1+\frac{1}{x}\right)dx$

$= \int x\log(1+x)\,dx - \int x\log x\,dx$

$= \left\{\frac{x^2}{2}\log(1+x) - \int \frac{x^2}{2(1+x)}dx\right\} - \left\{\frac{x^2}{2}\log x - \int \frac{1}{x}\times\frac{x^2}{2}dx\right\}$

$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\int \frac{x^2-1+1}{(1+x)}dx - \frac{x^2}{2}\log x + \frac{1}{4}x^2 + C$

$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\left\{\frac{x^2}{2} - x + \log(1+x)\right\}$
$- \frac{x^2}{2}\log x + \frac{1}{4}x^2 + C$

$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\log(1+x) + \frac{x}{2} - \frac{x^2}{2}\log x + C$

$= \left(\frac{x^2-1}{2}\right)\log(1+x) - \frac{x^2}{2}\log x + \frac{x}{2} + C$

Therefore, $f(x) = \frac{x^2-1}{2}$, $g(x) = -\frac{x^2}{2}$ and $L = \frac{1}{2}$.

4. (a) : Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$.

Now, $|A|^2 = |A|^2 = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}^2 = [5(5\alpha)]^2 = (25\alpha)^2$

$\because |A^2| = 25 \Rightarrow (25\alpha)^2 = 25 \Rightarrow \alpha^2 = \frac{1}{25}$ $\therefore |\alpha| = \frac{1}{5}$

5. (d) : We know that probability of any event lies between 0 and 1 and both 0 and 1 are inclusive.

$\therefore \quad 0 \le \frac{1+3p}{4} \le 1 \Rightarrow -\frac{1}{3} \le p \le 1$

$0 \le \frac{1-p}{3} \le 1 \Rightarrow -2 \le p \le 1$

$0 \le \frac{1-3p}{2} \le 1 \Rightarrow -\frac{1}{3} \le p \le \frac{1}{3}$

Since, the events are mutually exclusive, therefore

$0 \le \frac{1+3p}{4} + \frac{1-p}{3} + \frac{1-3p}{2} \le 1 \Rightarrow \frac{1}{13} \le p \le 1$

Thus, the set of values of p is $\left[\frac{1}{13}, \frac{1}{3}\right]$.

6. (c) : Here $|\vec{a}| = 4, |\vec{b}| = 2$

$\therefore |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$ [$\because |\hat{n}| = 1$ and $0 \le \theta \le \pi \therefore \sin\theta > 0$]

$\Rightarrow |\vec{a}\times\vec{b}|^2 = 4^2 \times 2^2 \times \sin^2\frac{\pi}{6} = 16\times 4\times\frac{1}{4} = 16$

$\therefore (\vec{a}\times\vec{b})^2 = 16$

7. (a) : Given that α and β are the roots of the equation $ax^2 + bx + c = 0$.

$\therefore \quad \alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2}{a^2} - 2\frac{c}{a} = \frac{b^2-2ac}{a^2}$

and $\alpha^2\beta^2 = (\alpha\beta)^2 = \frac{c^2}{a^2}$

$\therefore$ Required equation is $x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$

$\Rightarrow x^2 - \left(\frac{b^2-2ac}{a^2}\right)x + \frac{c^2}{a^2} = 0$

$\Rightarrow a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

8. (c) : We have, $f(x)=\begin{cases}\dfrac{\sin\{\cos x\}}{x-\dfrac{\pi}{2}}, & x\neq\dfrac{\pi}{2}\\ 1, & x=\dfrac{\pi}{2}\end{cases}$

Now, $\lim\limits_{x\to\frac{\pi}{2}^-} f(x)=\lim\limits_{h\to0}\left(\dfrac{\sin(\sin h)}{-h}\right)=-1$

Again, $\lim\limits_{x\to\frac{\pi}{2}^+} f(x)=\lim\limits_{h\to0}\left(\dfrac{\sin(1-\sin h)}{h}\right)\neq-1$

$\therefore \lim\limits_{x\to\frac{\pi}{2}^-} f(x)\neq\lim\limits_{x\to\frac{\pi}{2}^+} f(x)$

So, $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ does not exist.

9. (d): $\lim\limits_{h\to0}\dfrac{\{f(x+h)\}^2-\{f(x)\}^2}{2h}$

$=\dfrac{1}{2}\lim\limits_{h\to0}\{f(x+h)+f(x)\}\times\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{1}{2}\cdot2f(x)f'(x)=f(x)f'(x).$

10. (d): The required middle term is,

$t_{n+1}={}^{2n}C_n\cdot x^{2n-n}\cdot\left(\dfrac{1}{x}\right)^n={}^{2n}C_n x^n\cdot\dfrac{1}{x^n}$

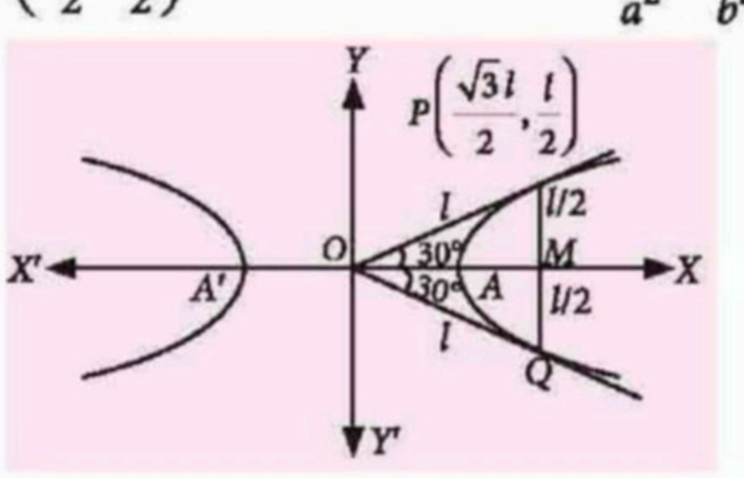
$={}^{2n}C_n=\dfrac{\lfloor 2n}{\lfloor n\,\lfloor n}=\dfrac{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot\ldots\ldots\cdot(2n-1)\cdot2n}{\lfloor n\,\lfloor n}$

$=\dfrac{(1\cdot3\cdot5\cdot\ldots\cdot(2n-1))(2\cdot4\cdot6\cdot\ldots\cdot2n)}{\lfloor n\,\lfloor n}$

$=\dfrac{(1\cdot3\cdot5\cdot\ldots\cdot(2n-1))2^n\,\lfloor n}{\lfloor n\,\lfloor n}=\dfrac{(1\cdot3\cdot5\cdot\ldots\cdot(2n-1))}{\lfloor n}\cdot2^n$

11. (d): Let the length of each side of the equilateral triangle OPQ be l units. Then, the coordinates of P are $\left(\dfrac{\sqrt{3}l}{2},\dfrac{l}{2}\right)$

Point $P\left(\dfrac{\sqrt{3}l}{2},\dfrac{l}{2}\right)$ lies on the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$



$\therefore \dfrac{3l^2}{4a^2}-\dfrac{l^2}{4b^2}=1\Rightarrow(3b^2-a^2)l^2=4a^2b^2$

$\Rightarrow (3b^2-a^2)l^2=4a^2b^2$

$\Rightarrow (3e^2-4)l^2=4a^2(e^2-1)$

$\Rightarrow l=2a\sqrt{\dfrac{e^2-1}{3e^2-4}}$

Since, l is real and $e>1$,

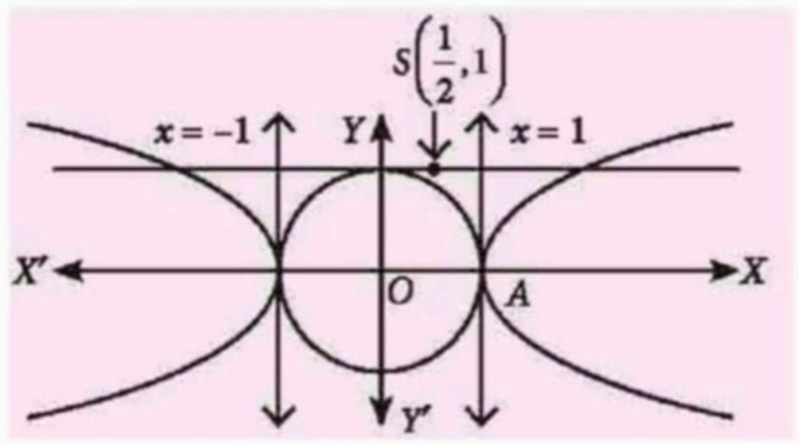
$\therefore\ 3e^2-4>0\Rightarrow e^2>\dfrac{4}{3}\Rightarrow e>\dfrac{2}{\sqrt{3}}$

12. (a) : For the circle $x^2+y^2=1$ and rectangular hyperbola $x^2-y^2=1$, one common tangent is evidently $x=1$, the other being $x=-1$.

We require the equation of ellipse in standard form with focus at $S\left(\dfrac{1}{2},1\right)$ and directrix $x=1$, which is

$\left(x-\dfrac{1}{2}\right)^2+(y-1)^2=\left(\dfrac{1}{2}\right)^2(x-1)^2$

$\Rightarrow \dfrac{3x^2}{4}-\dfrac{x}{2}+(y-1)^2=0$



$\Rightarrow \dfrac{3}{4}\left(x-\dfrac{1}{3}\right)^2+(y-1)^2=\dfrac{1}{12}$

$\Rightarrow 9\left(x-\dfrac{1}{3}\right)^2+12(y-1)^2=1$

13. (a) : Here, $3[x]-5\dfrac{|x|}{x}=\begin{cases}3[x]-5, & \text{if } x>0\\ 3[x]+5, & \text{if } x<0\end{cases}$

Now, $\int\limits_{-3/2}^{2} f(x)dx=\int\limits_{-3/2}^{-1} f(x)dx+\int\limits_{-1}^{0} f(x)dx+\int\limits_{0}^{1} f(x)dx+\int\limits_{1}^{2} f(x)dx$

$=\int\limits_{-3/2}^{-1}(3(-2)+5)dx+\int\limits_{-1}^{0}(-3+5)dx+\int\limits_{0}^{1}(0-5)dx+\int\limits_{1}^{2}(-2)dx$

$=-1\left(-1+\dfrac{3}{2}\right)+2(1)-5(1)+(-2)\cdot1$

$$=-\frac{1}{2}+2-5-2=-\frac{11}{2}$$

14. (a) : $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$

$$\Rightarrow \quad \frac{dp(t)}{dt}=\frac{p(t)-400}{2} \Rightarrow \int\frac{dp(t)}{p(t)-400}=\frac{1}{2}\int dt$$

$$\Rightarrow \quad \log|p(t)-400|=\frac{1}{2}t+\log c$$

$$\Rightarrow \quad \log\left|\frac{p(t)-400}{c}\right|=\frac{1}{2}t \Rightarrow p(t)=ce^{\frac{t}{2}}+400$$

At $p(0) = 100$, we get $100=ce^{\frac{0}{2}}+400 \Rightarrow c=-300$

$\therefore$ The required solution is, $p(t)=400-300e^{\frac{t}{2}}$

15. (c) : Clearly, $p(0 < x < 3) = p(x = 1) + p(x = 2)$

$$=\frac{{}^3C_1\times{}^7C_3}{{}^{10}C_4}+\frac{{}^3C_2\times{}^7C_2}{{}^{10}C_4}=\frac{4}{5}$$

16. (b) : $z=\dfrac{1+\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}}{1+\sin\frac{\pi}{3}-i\cos\frac{\pi}{3}}=\dfrac{1+\frac{\sqrt{3}}{2}+\frac{i}{2}}{1+\frac{\sqrt{3}}{2}-\frac{i}{2}}$

$$=\frac{2+\sqrt{3}+i}{2+\sqrt{3}-i}\times\frac{2+\sqrt{3}+i}{2+\sqrt{3}+i}=\frac{(3+2\sqrt{3})+i(2+\sqrt{3})}{3+2\sqrt{3}}$$

$$\therefore \quad \arg(z)=\tan^{-1}\left(\frac{2+\sqrt{3}}{3+2\sqrt{3}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$$

17. (c) :

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

$$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)$$

$$\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$$

$$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)=\sin^2\frac{\pi}{8}\cdot\sin^2\frac{3\pi}{8}$$

$$=\frac{1}{4}\left(2\sin^2\frac{\pi}{8}\right)\left(2\sin^2\frac{3\pi}{8}\right)=\frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$$

$$=\frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)=\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}$$

18. (d) : The relation R is defined by aRb if and only if $\sin^2 a + \cos^2 b = 1$

Now, $\sin^2 a + \cos^2 a = 1$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\therefore$ aRa holds. So R is reflexive.

Again, let aRb holds. $\therefore$ $\sin^2 a + \cos^2 b = 1$

$\Rightarrow 1 - \cos^2 a + 1 - \sin^2 b = 1 \Rightarrow \sin^2 b + \cos^2 a = 1$

$\therefore$ bRa holds when aRb holds. $\therefore$ R is symmetric.

Next, let aRb holds and bRc holds.

$\therefore$ $\sin^2 a + \cos^2 b = 1$ and $\sin^2 b + \cos^2 c = 1$

Now, $\sin^2 a + \cos^2 b + \sin^2 b + \cos^2 c = 1 + 1$

$\Rightarrow$ $\sin^2 a + \cos^2 c = 2 - 1 = 1$

$\therefore$ aRc holds when aRb and bRc hold.

$\therefore$ R is transitive.

As R is reflexive, symmetric and transitive, hence it is an equivalence relation.

19. (d) : Let, $n_1, n_2 \in N$ $\therefore$ $f(n_1) = f(n_2)$

$\Rightarrow$ $(n_1 + 5)^2 = (n_2 + 5)^2 \Rightarrow n_1 + 5 = n_2 + 5$

$[\because -(n_2 + 5) \notin N]$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

							9	
	7				3	5		
1			5				4	
				2				
	9			1				
				4		2		6

						1		
							4	3
						5		7
						9		
								1
						4	7	

						7	5	6
	8							
6	1	7	3					
	4				2			

2	1	8		3			6	
6			8			4		
7		4		5				1

						1	3	
						7		
						2		

						8	4	
						1		
						3		

							6	
						4		
						1		
						6		

				6		2	5	4
4	6				2			
7	2			1				
						8	3	
	8			2	5	6		7
			4				2	
5					6			9
3				7			1	2
2								

3								

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$\Rightarrow \quad n_1 = n_2$

$\therefore$ f is an one to one mapping.

Again $f(1) = (1 + 5)^2 = 36$, $f(2) = (2 + 5)^2 = 49$,

$f(3) = (3 + 5)^2 = 64$, ...

$\therefore$ Range of $f = \{36, 49, 64, ...\} \neq N$ = co-domain.

Therefore f is not onto mapping.

20. (a) : The equation of the line passing through $P(1, -2, 3)$ and parallel to the given line is

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6} \qquad ...(i)$$

Any point Q on (i) is given by $Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$.

This point Q will lie on $x - y + z = 5$.

Therefore, $2\lambda + 1 - (3\lambda - 2) + (-6\lambda + 3) = 5$

$$\Rightarrow \quad -7\lambda=-1 \Rightarrow \lambda=\frac{1}{7}$$

Thus, the coordinates of Q are

$$\left(\frac{2}{7}+1, \frac{3}{7}-2, -\frac{6}{7}+3\right)=\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$$

Hence, the required distance, $PQ=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=1$

21. (1) : We have, $\cos\left[2\tan^{-1}\frac{1+x}{1-x}+\sin^{-1}\frac{1-x^2}{1+x^2}\right]$

Let $x = \tan\alpha$

$$\therefore \quad \cos\left[2\tan^{-1}\frac{1+\tan\alpha}{1-\tan\alpha}+\sin^{-1}\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right]$$

$$=\cos\left[2\tan^{-1}\tan\left(\frac{\pi}{4}+\alpha\right)+\sin^{-1}\cos 2\alpha\right]$$

$$=\cos\left(\frac{\pi}{2}+2\alpha+\frac{\pi}{2}-2\alpha\right) \qquad \left(\because \cos 2\alpha=\sin\left(\frac{\pi}{2}-2\alpha\right)\right)$$

$= \cos\pi = -1$

$$\therefore \quad -\cos\left[2\tan^{-1}\frac{1+x}{1-x}+\sin^{-1}\frac{1-x^2}{1+x^2}\right]=-(-1)=1$$

22. (19) : Let the equations of AB, BC and AC of ΔABC are respectively :

$x + 8y - 22 = 0$ ...(i)

$5x + 2y - 34 = 0$ ...(ii)

$2x - 3y + 13 = 0$ ...(iii)

Now, solving (i) and (ii), we get (6, 2); solving (ii) and (iii), we get (4, 7) and solving (i) and (iii), we get (–2, 3)

$$\therefore \quad \text{Required area}=\left|\frac{1}{2}\{6(3-7)-2(7-2)+4(2-3)\}\right|$$

$$=\left|\frac{1}{2}(-24-10-4)\right|=|-19|=19 \text{ sq. units.}$$

23. (26) : Any tangent to

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \text{ is } y=mx\pm\sqrt{9m^2+4} \qquad ... (i)$$

It touches $\frac{x^2}{4}+\frac{y^2}{9}=1$

So, (i) is of the form $x=m'y\pm\sqrt{9m'^2+4}$ ...(ii)

Comparing (i) and (ii), $\frac{m}{-1}=\frac{-1}{m'}=\frac{\sqrt{9m^2+4}}{\sqrt{9m'^2+4}}$

$$\Rightarrow \quad mm'=1 \text{ and } m^2=\frac{9m^2+4}{9m'^2+4}$$

$\Rightarrow \quad m^2(9m'^2 + 4) = 9m^2 + 4$

$\Rightarrow \quad 9 + 4m^2 = 9m^2 + 4 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

Therefore the common tangents are

$$y=x\pm\sqrt{13} \text{ and } y=-x\pm\sqrt{13}$$

Clearly, the above 4 lines form a square of sides $\sqrt{26}$ units and area 26 square units.

24. (8) : Here $f(x) = ax^3 + bx^2 + cx$

$\therefore \quad f'(x) = 3ax^2 + 2bx + c$

Now, we have $f'(1) = 0 \Rightarrow 3a + 2b + c = 0$

and $f'(5) = 0 \Rightarrow 75a + 10b + c = 0$

[$\because$ $f(x)$ has extrema at $x = 1$ and $x = 5$]

Again, $\int_{-1}^{1} f(x)\,dx=\int_{-1}^{1}(ax^3+bx^2+cx)\,dx$

$$=\left[a\frac{x^4}{4}+b\frac{x^3}{3}+c\frac{x^2}{2}\right]_{-1}^{1}=b\left(\frac{1}{3}+\frac{1}{3}\right)=6$$

$$\Rightarrow \quad \frac{2b}{3}=6 \quad \therefore \quad b=9$$

Then, $0 = 3a + 18 + c$ and $0 = 75a + 90 + c$

$\therefore \quad 0 = 72a + 72 \Rightarrow a = -1$ and $c = -15$

Therefore, $a + b = 8$

25. (5) : We know, the variance of binomial distribution is, $\text{Var}(X) = npq$

$$=n\left[\left(\frac{p+q}{2}\right)^2-\left(\frac{p-q}{2}\right)^2\right]$$

$$=n\left[\frac{1}{4}-\left(\frac{p-q}{2}\right)^2\right]\le\frac{n}{4} \qquad \left[\because \left(\frac{p-q}{2}\right)^2\ge 0\right]$$

Hence, the variance of the binomial distribution can never exceed $n/4$. Hence, for $n = 20$, maximum value of $\text{Var}(X)$ is $20/4 = 5$.

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BRUSH UP

for

JEE Main & Advanced 2020

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Unit 10

Differential Equations

- An equation involving dependent variable, independent variable and derivative of dependent variable with respect to independent variable is called a differential equation. If x is independent variable, y is a variable depends on x, then equation of the form $F\left(x, y, \frac{dy}{dx}\right)=0$ or $F\left(x, \frac{dy}{dx}\right)=0$ is called differential equation.

  e.g. $\frac{dy}{dx}+y\cos x=\sin x$, $\frac{dy}{dx}=\sin x \log x$, etc.

- **Order and degree of a differential equation:** The order of a differential equation is the order of its highest order derivative and the exponent of highest order derivative (highest order differential coefficient) occurring in the differential equation providing the differential equation is free from all radicals and fractional powers is called degree of the differential equation.

  If differential equation is $\frac{dy}{dx}+\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=0$, then its order is 2 and degree is 2, as highest order derivative is $\frac{d^2y}{dx^2}$, whose exponent is also 2.

Types of Differential Equations

- **Ordinary differential equation :** A differential equation is called ordinary differential equation if the equation has single independent variable like $\frac{d^2y}{dx^2}+kx=0,\ k\in R.$

- **Linear differential equation :** A differential equation is said to be linear if the dependent variable and its differential coefficient always occurs in first degree and are not multiplied together. The linear differential equation of first order and first degree is given by $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only or constants and $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only or constants.

- **Homogeneous differential equation :** A differential equation of the form $\frac{dy}{dx}=\frac{f(x, y)}{\phi(x, y)}$, where $f(x, y)$ and $\phi(x, y)$ are functions of same degree is called homogeneous differential equation.

- **Exact differential equation :** A differential equation is said to be exact if it can be obtained from its solution by direct differentiation without any further transformation such as elimination etc.
  The equation $(x^2 - ay)dx + (y^2 - ax)dy = 0$ is an example of exact differential equation which can be derived from its solution $x^3 - 3axy + y^3 = k$ by direct differentiation. In other words, an equation $Mdx + Ndy = 0$ is said to be exact if $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

Formation of a Differential Equation

- A differential equation can be obtained from its solution by process of differentiation and other algebraic techniques like addition, subtraction, multiplication and elimination etc.

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Suppose the solution of a differential equation contains n arbitrary constants and we need the corresponding differential equation whose solution is given then we use the following steps to form the differential equation.

(1) Differentiate the given equation n times by which we get n additional equations containing n arbitrary constants and derivatives. In this way, we have one original equation and n other equations *i.e.*, in total we have $(n + 1)$ equations.

(2) Eliminate n arbitrary constants from these $(n + 1)$ equations, in this way we get an equation of n^{th} order which is the required differential equation.

Solution of a Differential Equation

- Solution of a differential equation is a relationship between the dependent and independent variables, free from the derivative of the dependent variable. In other words, solution of a differential equation means to obtain a family of curves with number of parameters equal to the order of the differential equation.

Types of Solutions

- **General solution (G.S.) :** A solution of differential equation is said to be general solution if number of arbitrary constants in the solution are equal to the order of the differential equation.
- **Particular solution (P.S.) :** A solution obtained from general solution (G.S.) by putting the particular values to arbitrary constants is called particular solution.

Methods of Solving Differential Equations

- **Method of variable separable :** If a differential equation can be expressed in the form

$$f(x)\,dx + f(y)\,dy = 0 \quad \text{... (i)}$$

then we say that the variables are separable and solution obtained on integrating (i) both sides. The solution of (i) is given by

$\int f(x)\,dx + \int f(y)\,dy = c$, where '$c$' is an arbitrary constant which can be chosen in any form suitable for the answer to be in the simplified form. This arbitrary constant c can be taken as $\log c$, e^c, $\sin c$, $\cos c$, $\tan^{-1} c$, etc.

- **Method of substitution :** Sometimes the equation is not in the form to separate the variables but some proper substitution reduces the equation in variable separable form in new variable.

If $\frac{dy}{dx} = f(ax+by+c)$ ... (i) then put $z = ax + by + c$

$\Rightarrow \frac{dz}{dx} = a + b\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{b}\left(\frac{dz}{dx} - a\right)$

$\therefore$ By (i), we have $\frac{dz}{dx} = bf(z) + a$

$\Rightarrow \frac{dz}{a+bf(z)} = dx$ ... (ii)

In (ii), the variables x and z are separated and solution is obtained on integrating (ii) both sides *i.e.*, the required solution is

$\int \frac{dz}{a+bf(z)} = x + k$, where $z = ax + by + c$, $k \in R$,

which is the general solution of the differential equation (i).

- **Method of solving homogeneous equation :**

Equations of the form $\frac{dy}{dx} = \frac{f(x, y)}{\phi(x, y)}$ ... (i)

where $f(x, y)$ and $\phi(x, y)$ are homogeneous functions of x and y of the same degree are called homogeneous equations. These differential equations, can be reduced to variable separable form by the substitution of $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Replace v by y/x in the solution.

- **Differential equation reducible to homogeneous form :** The equation of the form $\frac{dy}{dx} = \frac{ax+by+k_1}{Ax+By+k_2}$ ...(i)

$(aB \neq Ab)$can be reduced to homogeneous form by substituting $x = X + h$, $y = Y + k$ where h, k are constants to be chosen so as to make the given equation homogeneous, we have

$\frac{dY}{dX} = \frac{dy}{dx} = \frac{aX+bY+(ah+bk+k_1)}{AX+BY+(Ah+Bk+k_2)}$ ...(ii)

where h, k be chosen so as to satisfy the equations.

$ah + bk + k_1 = 0$ and $Ah + Bk + k_2 = 0$ ...(iii)

On solving (iii), we get

$h = \frac{bk_2 - Bk_1}{aB - Ab}$, $k = \frac{Ak_1 - ak_2}{aB - Ab}$ and $aB \neq Ab$

By using these values of h, k in equation (ii), we have $\frac{dY}{dX} = \frac{aX+bY}{AX+BY}$

which can be solved by using the substitution $Y = vX$ and then X and Y should be replaced by $x - h$ and $y - k$ respectively.

- **Method to solve the linear differential equation :**
The linear differential equation (first order and first degree) is $\frac{dy}{dx}+Py=Q$ ... (i)

where P and Q are functions of x only or constants.
To solve such equatins, we multiply both sides of (i) by $e^{\int Pdx}$, where $e^{\int Pdx}$ is known as integrating factor (I. F.).

$$e^{\int Pdx}\left(\frac{dy}{dx}+Py\right)=Qe^{\int Pdx}$$

$$\Rightarrow \frac{d}{dx}\left(y\cdot e^{\int Pdx}\right)=Q\,e^{\int Pdx} \quad ...(ii)$$

Integrating both sides of (ii), we get
$y\cdot e^{\int Pdx}=\int(Q\cdot e^{\int Pdx})\,dx+k$, which is the required solution of given differential equation.

Equation reducible to the linear form

- **Bernoulli's equation :** A differential equation of the form $\frac{dy}{dx}+Py=Q\cdot y^n$, where P and Q are functions of x only or constants is called a Bernoulli's equation.
On dividing the above equation by y^n, we get

$$y^{-n}\frac{dy}{dx}+y^{-(n-1)}P=Q$$

By putting $y^{-(n-1)}=v$ so that $\frac{(1-n)}{y^n}\frac{dy}{dx}=\frac{dv}{dx}$,

we get $\frac{dv}{dx}+(1-n)\,P\cdot v=(1-n)Q$, which is a linear differential equation and can be solved as above.

ORTHOGONAL TRAJECTORIES

The orthogonal trajectories of a family of curves form another family of curves in which each curve of one family intersect all the curves of other family at 90° angle.

Steps to find orthogonal trajectory

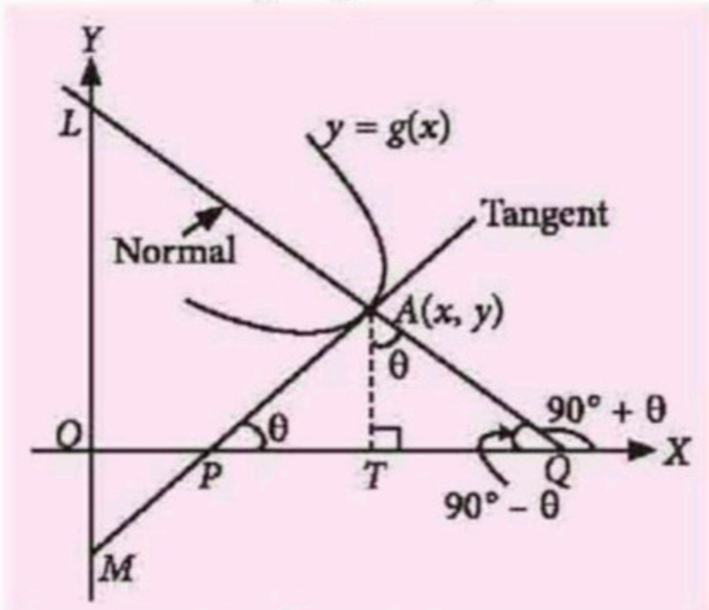
Let $f(x, y, c) = 0$ be the equation of the family of curves where c is an arbitrary constant or the parameter.

- Differentiate the equation $f(x, y, c) = 0$ with respect to x and eliminate the parameter c.
- Put $-\frac{dx}{dy}$ for $\frac{dy}{dx}$ in the differential equation so obtained which gives the differential equation of the orthogonal trajectories.
- After solving the differential equation, the result obtained is required orthogonal trajectory.

Note : If the orthogonal trajectories form the identical (same) family of the curves as the given family of curves then given system of curves is known as self orthogonal.

GEOMETRICAL APPLICATIONS

- Let $A(x, y)$ be any point on a continuous curve $y = g(x)$. Let tangent and normal at $A(x, y)$ meet X-axis at P and Q respectively.



Draw perpendicular AT from A on X-axis
i.e., $AT = y$.
If tangent at A makes angle θ with the positive direction of X-axis, then $\frac{dy}{dx}=\tan\theta$, known as slope of tangent. Slope of normal at $A(x, y)$ is

$$-\frac{1}{\text{slope of tangent at } A(x,y)}=-\left(\frac{dx}{dy}\right)_{A(x,y)}$$

- **Length of tangent :** From figure, AP is defined as the length of tangent. In ΔATP, we have

$$AP=|y\,\text{cosec}\,\theta|=\left|y\cdot\sqrt{1+\cot^2\theta}\right|$$

$$=\left|\frac{y\sqrt{(1+\tan^2\theta)}}{\tan\theta}\right|=\frac{y\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}}{\frac{dy}{dx}}$$

$$\text{or}\quad AP=\left|y\sqrt{\left(1+\frac{dx}{dy}\right)^2}\right|$$

- **Length of normal :** From figure, AQ is defined as length of normal. In ΔATQ, we have

$$AQ=|y\,\text{cosec}(90^\circ-\theta)|=|y\,\sec\theta|$$

$$=\left|y\sqrt{1+\tan^2\theta}\right|=\left|y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right|$$

- **Length of subtangent :** From figure, PT is defined as length of subtangent. In ΔAPT, we have

$$PT=|y\cot\theta|=\left|\frac{y}{\tan\theta}\right|=\left|y\frac{dx}{dy}\right|$$

= projection of tangent on X-axis

- **Length of subnormal :** From figure, TQ is defined as length of subnormal. In ΔATQ, we have

$$TQ = |y \cot(90° - \theta)| = |y \tan\theta| = \left| y \frac{dy}{dx} \right|$$

- **Intercept made by tangent on the coordinate axes:** From figure, we notice that intercept made by tangent on X-axis and Y-axis are OP and OM respectively. The equation of tangent at $A(x, y)$ is given by

$$Y - y = \frac{dy}{dx}(X - x) \quad ...(i)$$

Putting $Y = 0$ and $X = 0$ respectively in (i), we get

$$OP = x - y\frac{dx}{dy} \text{ and } OM = y - x\frac{dy}{dx}$$

- **Intercept made by normal on the coordinate axes :** From figure, we notice that intercept made by normal on X-axis and Y-axis are OQ and OL respectively. The equation of normal at $A(x, y)$ is given by

$$Y - y = -\frac{dx}{dy}(X - x) \quad ...(ii)$$

Putting $Y = 0$ and $X = 0$ respectively in (ii), we get

$$OQ = x + y\frac{dy}{dx} \text{ and } OL = y + x\frac{dx}{dy}$$

- For polar coordinates, we have the following results.

If $x = r\cos\theta$, $y = r\sin\theta$, then

(1) Length of radius vector, $r = \sqrt{x^2 + y^2}$

(2) $x dx + y dy = r dr$

(3) $x dy + y dx = r^2 d\theta$

(4) $dx^2 + dy^2 = dr^2 + r^2 d\theta$

CLAIRAUT'S FORM OF DIFFERENTIAL EQUATION

A differential equation of the form $y = Px + f(P)$, $\left(\text{where } P = \frac{dy}{dx}\right)$ is called Clairaut form of differential equation whose solution is obtained by replacing P by arbitrary constant C

$$y = Px + f(P) \quad ...(i)$$

On differentiating (i) with respect to x and putting $\frac{dy}{dx} = P$, we get

$$P = P + x\frac{dP}{dx} + f'(P)\frac{dP}{dx} \Rightarrow [x + f'(P)]\frac{dP}{dx} = 0$$

$$\therefore \quad \frac{dP}{dx} = 0 \Rightarrow P = C$$

$\therefore$ From (i), we get $y = Cx + f(C)$

FORMATION OF DIFFERENTIAL EQUATIONS

- If $y = A\ e^{ax} + B\ e^{bx} + C\ e^{cx}$ is the solution of a differential equation, then corresponding differential equation is given by $[(D - a)(D - b)(D - c)]y = 0$, where

$$D(y) = \frac{dy}{dx},\ D^2(y) = \frac{d^2y}{dx^2},\ \text{....so on .}$$

$$\Rightarrow [D^3 - D^2(a + b + c) + D(ab + bc + ca) + (-a)(-b)(-c)]y = 0$$

$$\Rightarrow \frac{d^3y}{dx^3} - (a+b+c)\frac{d^2y}{dx^2} + (ab+bc+ca)\frac{dy}{dx} + (-a)(-b)(-c)y = 0$$

- If $y = e^x(A\cos 2x + B\sin 2x)$ be the solution of a differential equation, then corresponding differential equation is given by $[(D - 1 + 2i)(D - 1 - 2i)]y = 0$

$$\Rightarrow (D^2 - 2D + 5)y = 0 \Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$$

Generally for $y = e^{kx}(A\cos mx + B\sin mx)$, the differential equation is given by

$$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + (k^2 + m^2)y = 0$$

- Let the solution of a differential equation is given in the form $y = (A + Bx)\ e^{kx}$, then corresponding differential equation is given by $(D - k)^2 y = 0$, (as in the solution there are 2 unknowns like A and B)

$$\Rightarrow D^2(y) - 2kD(y) + k^2y = 0 \Rightarrow \frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + k^2y = 0$$

- If $y = (A + Bx + Cx^2)\ e^{-kx}$ be the solution of a differential equation, then differential equation is given by $(D + k)^3 y = 0$

$$\text{i.e., } \frac{d^3y}{dx^3} + 3k\frac{d^2y}{dx^2} + 3k^2\frac{dy}{dx} + k^3y = 0$$

PROBLEMS

Single Option Correct Type

1. The differential equation of the family of curves $y = e^{2x}(A\cos 3x + B\sin 3x)$, where A and B are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

(b) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

(c) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 5y = 0$ (d) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 13y = 0$

2. The order and degree of the differential equation whose solution is $y = mx + m^3 - 3m^{3/2} + 4$, where m is arbitrary constant, are respectively

(a) 2 and 6 (b) 1 and 6
(c) 6 and 1 (d) 1 and 4

3. If $y = e^{3x} + 4e^{-5x}$ satisfies the relation $y_3 + Ay_1 + By = 0$, then value of $(A + B)$ equals

(a) -11 (b) 11 (c) 30 (d) -19

4. The solution of $x^3\frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying $y(1) = 0$ is $\sin y = e^x(x - 1)x^{-k}$, then the value of k equals

(a) 1 (b) 2 (c) 3 (d) 4

5. The differential equation whose solution represents the family $xy = Ae^{\lambda x} + Be^{-\lambda x}$, is

(a) $xy_2^2 + 2y_1 = xy$ (b) $xy_2^2 + 2y_1 = \lambda^2 xy$
(c) $xy_2 + 2y_1 = \lambda^2 xy$ (d) none of these

6. The curve passing through orgin and orthogonal to hyperbola $xy = 4$ is

(a) always decreasing function
(b) always increasing function
(c) partially increasing and decreasing function
(d) none of these

7. The solution of differential equation

$\frac{dy}{dx} = \frac{y - 5x^6 y}{x^7 + x + 4x^2 y^3}$, is (where c is a constant)

(a) $x^4 y + \frac{y}{x} + y^4 = c$ (b) $x^5 y + \frac{y}{x} + y^4 = c$

(c) $xy^3 + \frac{x}{y} + y^4 = c$ (d) $x^5 y^5 + \frac{y}{x} + y^4 = c$

8. The solution of differential equation $2x^3 y dy + (1 - y^2)(x^2 y^2 + y^2 - 1)dx = 0$, is (where k is a constant)

(a) $x^2 y^2 = (kx - 1)(1 - y^2)$
(b) $x^2 y^2 = (kx + 1)(1 + y^2)$
(c) $x^2 y^2 = (kx + 1)(1 - y^2)$
(d) $x^2 y^2 = (kx - 1)(1 + y^2)$

9. At each point (x, y) of the curve, intercept of the tangent on y-axis is equal to $2xy^2$, then equation of curve is

(a) $3x - x^2 y = cy, c \in R$ (b) $x + x^2 y = cx, c \in R$
(c) $y + xy^2 = cx, c \in R$ (d) $x - x^2 y = cy, c \in R$

10. The normal to the curve at a point $A(x, y)$ meet the x-axis at point B. If the distance of B from origin is double the abscissa of A, then curve represent itself a/an

(a) circle (b) ellipse
(c) hyperbola (d) parabola

More Than One Option(s) Correct Type

11. The orthogonal trajectories of $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$ (where λ is the arbitrary parameter) is given by $x^{k_1} + y^{k_2} = k_3(a^{k_4} \log x + 1)$ whenever the orthogonal trajectory passes through (1, 1), then

(a) $k_1 \cdot k_2 \cdot k_3 \cdot k_4 = k_1^4$ (b) $k_1 \cdot k_2 \cdot k_3 \cdot k_4 = k_2^4$
(c) $k_1 = k_2 = k_3 = k_4$ (d) $k_1 + k_3 = k_2 + k_4$

12. Let $f: [1, \infty) \to [2, \infty)$ be a differentiable function satisfying $f(1) = 2$. If $6\int_1^x f(t)dt = 3xf(x) - x^3 \ \forall\, x \geq 1$, then

(a) $f(2) = 6$ (b) $f(-2) = 2$
(c) $f(3) = 12$ (d) $f'(2) = 5$

13. A curve is such that mid point of the portion of tangent intercepted between the point where the tangent is drawn and the point where the tangent meets at y-axis lies on the line $y = 3x$. If the curve passes through $(-3, 0)$, then

(a) curve is a parabola whose focus is $\left(-\frac{3}{2}, -2\right)$
(b) curve is a parabola whose vertex is $\left(-\frac{3}{2}, -\frac{9}{4}\right)$
(c) curve is an ellipse whose equation of directrix is $2y + 5 = 0$
(d) origin satisfies the equation of curve

14. A normal is drawn at a point $A(x, y)$ to a curve which intersect the x-axis at B. If AB is of fixed length 'a', then

(a) the differential equation of the curve is $y\frac{dy}{dx} = \pm\sqrt{a^2 - y^2}$
(b) the curve represent a circle centred at origin
(c) the curve passes through $(a, 0)$
(d) the point $(0, a)$ lies on the curve

15. The curve for which the length of normal is equal to the length of the radius vector is

(a) a circle centred at origin
(b) a straight line
(c) a hyperbola (d) an ellipse

Monthly Test Drive CLASS XII ANSWER KEY

1. (d)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)
6. (b)	**7.** (a, c, d)	**8.** (b, d)	**9.** (b, c, d)	**10.** (b)
11. (a, c)	**12.** (a, c)	**13.** (a, b, c)	**14.** (c)	**15.** (b)
16. (c)	**17.** (6)	**18.** (2)	**19.** (3)	**20.** (3)

16. A curve passes through the point $A(3, 0)$ and slope of tangent at a point $P(x, 4)$ is given by $\frac{(x-4)^2+(y-4)}{x-4}$, then

(a) curve is a upward parabola

(b) the vertex of the curve is at $\left(\frac{3}{2}, -\frac{9}{4}\right)$

(c) curve is partially increasing and decreasing function

(d) the area bounded by the curve and x-axis in the fourth quadrant is $\frac{9}{2}$ sq. units.

17. The solution of differential equation

$\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$, is

(a) $y(1 - \cos x) = c$ (b) $y(1 + \cos x) = c$

(c) $x = 2\cos^{-1}\sqrt{\frac{c}{2y}}$ (d) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$

Comprehension Type

Paragraph for Q.No. 18 and 19

The differential equation corresponding to $y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$, where C_1, C_2, C_3 are arbitrary constants and m_1, m_2, m_3 are roots of the equation $m^3 - 9m^2 + 23m - 15 = 0$ is

$A\frac{d^3y}{dx^3} + B\frac{d^2y}{dx^2} + C\frac{dy}{dx} + Dy = 0$, where A, B, C, D are constants.

18. The sum of order and degree of differential equation is

(a) 4 (b) 3

(c) 2 (d) none of these

19. The value of $A + B + C + D$ is equal to

(a) 1 (b) 0 (c) 4 (d) 3

Matrix Match Type

20. Match the following :

Column-I		Column-II	
P.	The differential equation is obtained from its solution $y = a_1 e^{nx} + a_2\cos nx + a_3\sin nx$, n is known constant and a_1, a_2, a_3 being arbitrary constants. If O and D respectively the order and degree of differential equation, then	1.	$O + 2D = 5$
Q.	The differential equation is formed by differentiating and eliminating the unknown quantities a, b, c and d from $y = a\sin^2 x + b\cos^2 x + c\sin 2x + d\cos 2x$. If O is the order and D is the degree of differential equation, then	2.	$2O + 3D = 5$
R.	Order and degree of differential equation satisfying $\sqrt{1+x^2} + \sqrt{1+y^2} = A\left(x\sqrt{1+y^2} + y\sqrt{1+x^2}\right)$ are O and D, then	3.	$O = D$
		4.	$O^D + D^O = 2$
		5.	$2^O + 3^D = 11$

Numerical Value Type

21. If the differential equation whose solution is $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$, where A and B are arbitrary constants, is $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = \lambda$, then $\lambda^2 =$ _____.

22. If the solution of the differential equation $\frac{dy}{dx} + \frac{\cos x(3\cos y - 7\sin x - 3)}{\sin y(3\sin x - 7\cos y + 7)} = 0$ is $(\sin x + \cos y - 1)^{\lambda_1} \times (\sin x - \cos y + 1)^{\lambda_2} = c$, where c is arbitrary constant, then $(\lambda_1 + \lambda_2) =$ _____.

23. The differential equation whose solution is $y = (A + Bx + Cx^2)e^{-2x}$ is $y_3 + k_1y_2 + k_2y_1 + k_3y = 0$, then $k_1 + k_2 + k_3 =$ _____.

24. If the solution of the differential equation $y^2(y - xp) = x^4p^2$ is in the form $\frac{1}{y} = \frac{c}{x} + c^\lambda$, then value of λ equals to $\left(\text{where } p = \frac{dy}{dx} \text{ and } c \text{ is an arbitrary constant}\right)$ _____.

25. If the solution of the differential equation $y = 2xp - y^2p^3$ is in the form $y^2 = 2c^m x - c^n$, (where $p = \frac{dy}{dx}$ and c is an arbitrary constant), then $m + n =$ _____.

SOLUTIONS

1. (a) : Given, $y = e^{2x}(A\cos 3x + B\sin 3x)$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{dy}{dx} = 2\cdot e^{2x}(A\cos 3x + B\sin 3x)$

$+ e^{2x}(-3A\sin 3x + 3B\cos 3x)$

$$\Rightarrow \quad \frac{dy}{dx}-2y=e^{2x}(-3A\sin 3x+3B\cos 3x) \qquad ...(ii)$$
[Using (i)]

$$\Rightarrow \quad \frac{d^2y}{dx^2}-2\frac{dy}{dx}=2e^{2x}(-3A\sin 3x+3B\cos 3x)$$
$$+ e^{2x}(-9A\cos 3x - 9B\sin 3x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}-2\frac{dy}{dx}=2\left(\frac{dy}{dx}-2y\right)+(-9y) \text{ [Using (i) and (ii)]}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$$

2. (b) : Given, $y = mx + m^3 - 3m^{3/2} + 4$ ...(i)

$$\Rightarrow \quad \frac{dy}{dx}=m$$

From (i), $y=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3-3\left(\frac{dy}{dx}\right)^{3/2}+4$

$$\Rightarrow 3\left(\frac{dy}{dx}\right)^{3/2}=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3+4-y$$

$$\Rightarrow \quad 9\left(\frac{dy}{dx}\right)^3=\left(x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3+4-y\right)^2$$

$\therefore$ Order = 1, degree = 6.

3. (b) : We have, $y = e^{3x} + 4e^{-5x}$

$$\Rightarrow \quad y_1=3e^{3x}-20e^{-5x} \quad \Rightarrow \quad y_2 = 9e^{3x} + 100\, e^{-5x}$$

$$\Rightarrow \quad y_3=27e^{3x}-500e^{-5x}$$

Now, $y_3 + Ay_1 + By = 0$ [Given]

$\Rightarrow$ $(27e^{3x} - 500e^{-5x}) + A(3e^{3x} - 20e^{-5x})$
$+ B(9^{3x} + 4e^{-5x}) = 0$

$\Rightarrow$ $e^{3x}(27 + 3A + B) + e^{-5x}(-500 - 20A + 4B) = 0$

$\Rightarrow$ $3A + B = -27$ and $20A - 4B = -500$

On solving, we get $A = -19$ and $B = 30$

$\therefore$ $A + B = 11$

4. (d) : We have, $x^3\frac{dy}{dx}+4x^2\tan y=e^x\sec y$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{4\tan y}{x}=\frac{e^x\sec y}{x^3} \Rightarrow \quad \cos y\frac{dy}{dx}+\frac{4\sin y}{x}=\frac{e^x}{x^3}$$

$$\Rightarrow \quad \frac{dz}{dx}+\frac{4}{x}z=\frac{e^x}{x^3} \qquad \text{(By putting } \sin y=z\text{)}$$

Now, I.F. $= e^{\int\frac{4}{x}dx}=e^{4\log x}=x^4$

$\therefore$ Required solution is $zx^4=\int\left(\frac{e^x}{x^3}\cdot x^4\right)dx$

$\Rightarrow$ $z\cdot x^4=\int xe^x dx \Rightarrow zx^4 = e^x(x - 1) + C$

$\Rightarrow$ $zx^4 = e^x(x - 1)$ ($\because C = 0$ when $x = 1, z = 0$)

$\Rightarrow$ $\sin y = e^x (x - 1)x^{-4}$

Also, $\sin y = e^x(x - 1)x^{-k}$ [Given]

$\therefore$ $k = 4$

5. (c) : Given $xy = Ae^{\lambda x} + Be^{-\lambda x}$ ...(i)

Differentiating (i) w.r.t. x, we get

$y + xy_1 = \lambda(Ae^{\lambda x} - Be^{-\lambda x})$ ...(ii)

Differentiating (ii) w.r.t. x, we get

$y_1 + xy_2 + y_1 = \lambda^2(Ae^{\lambda x} + Be^{-\lambda x})$

$\Rightarrow$ $2y_1 + xy_2 = \lambda^2 xy$ (Using (i))

6. (b) : Let $m_1=\frac{dy}{dx}$ be the slope of orthogonal curve at point (x, y) and $m_2=\frac{dy}{dx}$ be the slope of hyperbola $xy = 4$ *i.e.*, $y=\frac{4}{x}$

$$\therefore \quad m_2=\frac{dy}{dx}=-\frac{4}{x^2}$$

Since the curves are orthogonal.

$$\therefore \quad m_1m_2=-1 \quad \Rightarrow \quad \frac{dy}{dx}\left(-\frac{4}{x^2}\right)=-1 \quad \Rightarrow \quad \frac{dy}{dx}=\frac{x^2}{4}>0$$

$\therefore$ Curve is always increasing function.

7. (b) : The differential equation can be written as

$(x^7 + x + 4x^2y^3)dy = (y - 5x^6y)dx$

$$\Rightarrow \quad \left(\frac{x^7+x+4y^3x^2}{x^2}\right)dy=\frac{y-5x^6y}{x^2}dx$$

$$\Rightarrow \quad x^5dy+5x^4ydx+\frac{xdy-ydx}{x^2}+4y^3dy=0$$

$$\Rightarrow \quad d(x^5y)+d\left(\frac{y}{x}\right)+d(y^4)=0$$

On integrating, we get $x^5y+\frac{y}{x}+y^4=c$

8. (a) : The differential equation can be written as

$$\frac{2y}{(1-y^2)^2}\cdot\frac{dy}{dx}+\frac{y^2}{(1-y^2)}\frac{1}{x}=\frac{1}{x^3} \quad \Rightarrow \quad \frac{dz}{dx}+\frac{z}{x}=\frac{1}{x^3}$$

[Putting $\frac{y^2}{1-y^2}=z$, we get $\frac{2y}{(1-y^2)^2}\frac{dy}{dx}=\frac{dz}{dx}$]

Now, I.F. $=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

So, solution is $z\cdot x=\int\frac{1}{x^3}\cdot xdx \quad \Rightarrow \quad z\cdot x=-\frac{1}{x}+k$

$\Rightarrow$ $zx^2 = (kx - 1) \Rightarrow \frac{y^2}{1-y^2}\cdot x^2=(kx-1)$

$\Rightarrow$ $x^2y^2 = (kx - 1)(1 - y^2)$

9. (d) : Equation of tangent at (x, y) is $Y-y=\frac{dy}{dx}(X-x)$

Now, intercept of the tangent on y-axis $=y-x\frac{dy}{dx}$

According to question

$$y - x\frac{dy}{dx} = 2xy^2 \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x\cdot y} = 2$$

$$\Rightarrow \frac{dz}{dx} + \frac{1}{x}\cdot z = 2 \quad \left(\text{Putting } \frac{1}{y} = z \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}\right)$$

Now, I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

So, solution is $z \cdot x = \int 2x\,dx$

$\Rightarrow z \cdot x = x^2 + c \Rightarrow \frac{x}{y} = x^2 + c$

$\Rightarrow x - x^2y = cy, c \in R$

10. (c) : Equation of normal at $A(x, y)$ is

$Y - y = -\frac{dx}{dy}(X - x)$, which meets at x-axis.

Now, intercept of the normal on x-axis $= x + y\frac{dy}{dx}$.

According to question, $OB = 2x$

$\Rightarrow x + y\frac{dy}{dx} = 2x \Rightarrow ydy = xdx$

$\Rightarrow y^2 - x^2 = c$, which is a hyperbola.

11. (a, b, c, d) : The given trajectory is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1 \quad ...(i)$$

$$\Rightarrow \frac{y}{a^2+\lambda} = \frac{1}{y}\left(\frac{a^2 - x^2}{a^2}\right) \quad ...(ii)$$

Differentiating (i) w.r.t. x, we get

$$\frac{2x}{a^2} + \frac{2y}{a^2+\lambda}\frac{dy}{dx} = 0 \Rightarrow \frac{x}{a^2} + \frac{y}{a^2+\lambda}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x}{a^2} + \frac{a^2 - x^2}{a^2 y}\frac{dy}{dx} = 0 \quad \text{[Using (ii)]} \quad ....(iii)$$

$\therefore$ Differential equation of orthogonal trajectory is

$$\frac{x}{a^2} - \frac{(a^2 - x^2)}{a^2 y}\frac{dx}{dy} = 0 \quad \left(\text{Replacing } \frac{dy}{dx} \text{ by } \frac{-dx}{dy} \text{ in (iii)}\right)$$

$$\Rightarrow ydy + \frac{x^2 - a^2}{x}dx = 0 \Rightarrow ydy + \left(x - \frac{a^2}{x}\right)dx = 0$$

$$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} - a^2\log x = C \Rightarrow C = 1$$

[Since, orthogonal trajectory passes through (1, 1)]

$\Rightarrow x^2 + y^2 - 2(1 + a^2 \log x) = 0$

Also, $x^{k_1} + y^{k_2} = k_3(1 + a^{k_4}\log x) = 0$ [Given]

$\Rightarrow k_1 = k_2 = k_3 = k_4 = 2$

12. (a, b, c, d) : Given, $6\int_1^x f(t)dt = 3xf(x) - x^3$

$\Rightarrow 6f(x) = 3[xf'(x) + f(x)] - 3x^2 \Rightarrow f(x) = xf'(x) - x^2$

$\Rightarrow y = x\frac{dy}{dx} - x^2$, where $y = f(x) \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = x$

Now, I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

So, solution is $y \cdot \frac{1}{x} = \int\left(x \cdot \frac{1}{x}\right)dx = x + C$

$\Rightarrow y = x^2 + Cx$

$\Rightarrow f(x) = x^2 + x$ ($\because C = 1$, when $x = 1, y = 2$)

13. (a, b, d) : Let (x, y) be the point on the curve and tangent drawn through this point meets y-axis.

The equation of tangent to the curve is $Y - y = \frac{dy}{dx}(X - x)$

The coordinates of the point where it meets the y-axis is $\left(0, y - x\frac{dy}{dx}\right)$.

Now mid point of (x,y) and $\left(0,\ y - x\frac{dy}{dx}\right)$ is

$\left(\frac{x}{2}, y - \frac{x}{2}\frac{dy}{dx}\right)$ which lies on the line $y = 3x$.

$\therefore \quad y - \frac{x}{2}\frac{dy}{dx} = 3\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} - \frac{2}{x}y = -3$, which is a linear differential equation.

Now, I.F. $= e^{-\int \frac{2}{x}dx} = \frac{1}{x^2}$

$\therefore$ Required solution is $y \cdot \frac{1}{x^2} = \int(-3)\frac{1}{x^2}dx = \frac{3}{x} + C$

$\Rightarrow y = 3x + Cx^2$

$\Rightarrow y = x^2 + 3x$ ($\because$ $C = 1$ as $(-3, 0)$ lies on it)

It is a parabola through origin.

$$\Rightarrow y + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2 \Rightarrow X^2 = 4\left(\frac{1}{4}\right)Y \quad ...(i)$$

where $X = x + \frac{3}{2}, Y = y + \frac{9}{4}$

So, coordinates of focus of parabola (i) are $\left(0, \frac{1}{4}\right)$.

$\therefore \quad x + \frac{3}{2} = 0$ and $y + \frac{9}{4} = \frac{1}{4}$

$\Rightarrow x = \frac{-3}{2}$ and $y = \frac{1}{4} - \frac{9}{4} = -2$

$\therefore$ Focus of the required parabola is $\left(-\frac{3}{2}, -2\right)$.

Vertex of parabola (i) is $(0, 0) \Rightarrow x + \frac{3}{2} = 0, y + \frac{9}{4} = 0$

$\therefore$ Vertex of required parabola is $\left(-\frac{3}{2}, -\frac{9}{4}\right)$.

14. (a, b, c, d): Equation of normal to the curve at $A(x, y)$ is given by $(Y-y)=-\frac{dx}{dy}(X-x)$ ...(i)

Let the coordinates of B are $(x_1, 0)$. Since (i) passes through $(x_1, 0)$.

$\therefore \quad (0-y)\frac{dy}{dx}=x-x_1 \Rightarrow y\frac{dy}{dx}=x_1-x$ ...(ii)

It is given that $AB = a$

$\therefore \quad (x-x_1)^2+(y-0)^2=a^2 \Rightarrow x-x_1=\pm\sqrt{a^2-y^2}$

$\Rightarrow \quad y\frac{dy}{dx}=\pm\sqrt{a^2-y^2}$ (Using(ii))

$\Rightarrow \quad \frac{ydy}{\sqrt{a^2-y^2}}=\pm dx$

Integrating, we get $\sqrt{a^2-y^2}=\pm x$

$\Rightarrow a^2 - y^2 = x^2 \Rightarrow x^2 + y^2 = a^2$, which represents a circle centred at origin and points $(a, 0)$, $(0, a)$ satisfies the equation of circle.

15. (a, c): Length of normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Length of radius vector $(r) = \sqrt{x^2+y^2}$,
where $x = r\cos\theta$, $y = r\sin\theta$

Given, $y\cdot\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\sqrt{x^2+y^2}$

$\Rightarrow \quad y^2\left(\frac{dy}{dx}\right)^2=x^2 \Rightarrow y\frac{dy}{dx}=x \text{ or } y\frac{dy}{dx}=-x$

$\Rightarrow ydy - xdx = 0$ or $ydy + xdx = 0$

$\Rightarrow y^2 - x^2 = k_1$ or $y^2 + x^2 = k_2$, which are respectively hyperbola and circle.

16. (a, b, c, d): The slope of the tangent to the curve is $\frac{dy}{dx}=\frac{(x-4)^2+y-4}{x-4}=x-4+\frac{y-4}{x-4}$

$\Rightarrow \quad \frac{dv}{du}=u+\frac{v}{u}$, where $u=x-4$ and $v=y-4$

$\Rightarrow \quad \frac{dv}{du}-\frac{v}{u}=u$, which is a linear differential equation.

Now, I.F. $= e^{-\int\frac{1}{u}du}=e^{-\log u}=\frac{1}{u}$

So, solution is $v\cdot\frac{1}{u}=\int\left(u\frac{1}{u}\right)du=u+C$

$\Rightarrow \quad v = u^2 + Cu \Rightarrow (y - 4) = (x - 4)^2 + C(x - 4)$

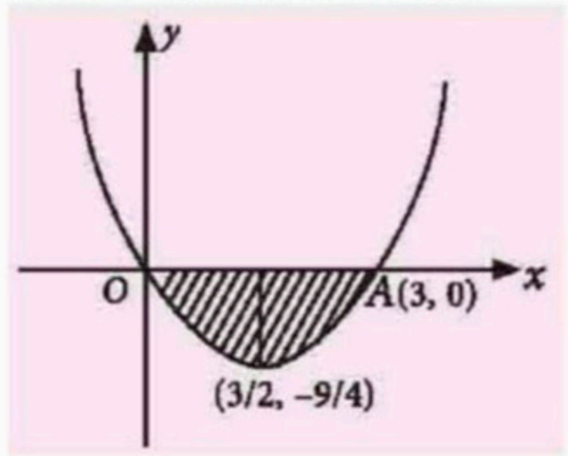
$\Rightarrow \quad y - 4 = (x - 4)^2 + 5(x - 4)$

[$\because C = 5$ as curve passes through (3, 0)]

$\Rightarrow \quad y = x^2 - 3x$

$\therefore$ Vertex of the curve is $\left(\frac{3}{2}, -\frac{9}{4}\right)$.

Also, curve is a monotonic function.



Area bounded by the curve $=\int_0^3(3x-x^2)dx=\frac{9}{2}$ sq. units

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17. (a, b, c, d) : We have, $\left(\frac{dy}{dx}\right)^2+2y\cot x\frac{dy}{dx}-y^2=0$

$$\therefore \quad \frac{dy}{dx}=\frac{-2y\cot x\pm\sqrt{4y^2\cot^2 x+4y^2}}{2}$$

$$=\frac{-2y\cot x\pm 2y\,\mathrm{cosec}x}{2}$$

$$\Rightarrow \quad \frac{dy}{dx}=y(-\cot x\pm\mathrm{cosec}x)=y\left(\frac{-\cos x\pm 1}{\sin x}\right)$$

$$=-y\left(\frac{\cos x\pm 1}{\sin x}\right)$$

$\Rightarrow \quad \frac{dy}{dx}+y\left(\frac{\cos x\pm 1}{\sin x}\right)=0$, which is a linear differential equation.

Now, I.F. $= e^{\int\left(\frac{\cos x\pm 1}{\sin x}\right)dx} = e^{\log(\sin x)\pm\log(\mathrm{cosec}x-\cot x)}$

$= \sin x(\mathrm{cosec}x-\cot x)$ and $\frac{\sin x}{\mathrm{cosec}x-\cot x}$

$= 1-\cos x$ and $(1+\cos x)=(1\pm\cos x)$

So, solution is $y(1\pm\cos x)=c$

$\Rightarrow \quad y(1+\cos x)=c$ or $y(1-\cos x)=c$

$\Rightarrow \quad y\left(2\cos^2\frac{x}{2}\right)=c$ or $y\left(2\sin^2\frac{x}{2}\right)=c$

$\Rightarrow \quad \cos^2\frac{x}{2}=\frac{c}{2y}$ or $\sin^2\frac{x}{2}=\frac{c}{2y}$

$\Rightarrow \quad \frac{x}{2}=\cos^{-1}\sqrt{\frac{c}{2y}}$ or $\frac{x}{2}=\sin^{-1}\sqrt{\frac{c}{2y}}$

$\Rightarrow \quad x=2\cos^{-1}\sqrt{\frac{c}{2y}}$ or $x=2\sin^{-1}\sqrt{\frac{c}{2y}}$

(18-19): Since m_1, m_2, m_3 are roots of the equation $m^3-9m^2+23m-15=0$.

$\Rightarrow \quad (m-1)(m-3)(m-5)=0$

$\Rightarrow \quad m_1=1,\ m_2=3,\ m_3=5$ ....(i)

$\therefore \quad y=C_1e^x+C_2e^{3x}+C_3e^{5x}$

$\therefore$ Required differential equation is

$[(D-1)(D-3)(D-5)]y=0$

$\Rightarrow \quad \frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$

$=A\frac{d^3y}{dx^3}+B\frac{d^2y}{dx^2}+C\frac{dy}{dx}+Dy$

18. (a) : Order = 3, degree = 1

$\therefore$ Sum of order and degree = 3 + 1 = 4

19. (b) : $A+B+C+D=1-9+23-15=0$

20. $P\rightarrow(1,5), Q\rightarrow(1,5), R\rightarrow(2,3,4)$

(P) $y=a_1e^{nx}+a_2\cos nx+a_3\sin nx$ ...(i)

$\therefore \quad y_1=na_1e^{nx}-na_2\sin nx+na_3\cos nx$ ...(ii)

$y_2=n^2a_1e^{nx}-n^2(a_2\cos nx+a_3\sin nx)$

$\Rightarrow \quad y_2=n^2a_1e^{nx}-n^2(y-a_1e^{nx})$ (Using (i))

$\Rightarrow \quad y_2=n^2a_1e^{nx}+n^2(a_1e^{nx}-y)$

$\Rightarrow \quad y_2=2n^2a_1e^{nx}-n^2y$ ... (iii)

$y_3=2n^3a_1e^{nx}-n^2y_1$

$=n(y_2+n^2y)-n^2y_1$ (Using (iii))

$\therefore \quad y_3-ny_2+n^2y_1-n^3y=0$

i.e., $\frac{d^3y}{dx^3}-n\frac{d^2y}{dx^2}+n^2\frac{dy}{dx}-n^3y=0 \quad \therefore \ O=3, D=1$

So, $O+2D=3+2(1)=5,\ 2^O+3^D=2^3+3^1=11$

(Q) $y=\frac{a(1-\cos 2x)}{2}+\frac{b(1+\cos 2x)}{2}+c\sin 2x+d\cos 2x$

$\therefore \quad y=\left(\frac{a}{2}+\frac{b}{2}\right)+c\sin 2x+\cos 2x\left(-\frac{a}{2}+\frac{b}{2}+d\right)$

$\Rightarrow \quad y=A+C\sin 2x+B\cos 2x,$

where $A=\frac{a}{2}+\frac{b}{2}, B=\frac{-a}{2}+\frac{b}{2}+d$ and $C=c$

Now, $y_1=2C\cos 2x-2B\sin 2x$ ...(i)

$y_2=-4C\sin 2x-4B\cos 2x$

$y_3=-8C\cos 2x+8B\sin 2x$

$=-4(2C\cos 2x-2B\sin 2x)$

$\Rightarrow \quad y_3=-4y_1$ (Using (i))

$\Rightarrow \quad \left(\frac{d^3y}{dx^3}\right)^1+4\frac{dy}{dx}=0 \quad \therefore \ O=3,\ D=1$

So, $O+2D=5,\ 2^O+3^D=2^3+3^1=11$

(R) Given, $\sqrt{1+x^2}+\sqrt{1+y^2}=A\left(x\sqrt{1+y^2}+y\sqrt{1+x^2}\right)$ ... (i)

Putting $x=\tan\theta$, $y=\tan\phi$ in (i), we get

$\sec\theta+\sec\phi=A(\tan\theta\sec\phi+\tan\phi\sec\theta)$

$\Rightarrow \quad \frac{\cos\theta+\cos\phi}{\cos\theta\cdot\cos\phi}=A\left(\frac{\sin\theta+\sin\phi}{\cos\theta\cdot\cos\phi}\right)$

$\Rightarrow \quad 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}=A\left(2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}\right)$

$\Rightarrow \quad \cot\frac{\theta+\phi}{2}=A \ \Rightarrow \ \theta+\phi=2\cot^{-1}A$

$\Rightarrow \quad \tan^{-1}x+\tan^{-1}y=2\cot^{-1}A$

On differentiating, we have $\frac{1}{1+x^2}+\frac{1}{1+y^2}y_1=0$

$\therefore \quad O=1, D=1$

So, $O=D, O^D+D^O=1+1=2,$

$2O+3D=2(1)+3(1)=5$

21. (4) : Given, $y=(\sin^{-1}x)^2+A\cos^{-1}x+B$

$\Rightarrow \quad (\sin^{-1}x)^2=y-A\cos^{-1}x-B$ ... (i)

Now, $\frac{dy}{dx}=2(\sin^{-1}x)\cdot\frac{1}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}}$

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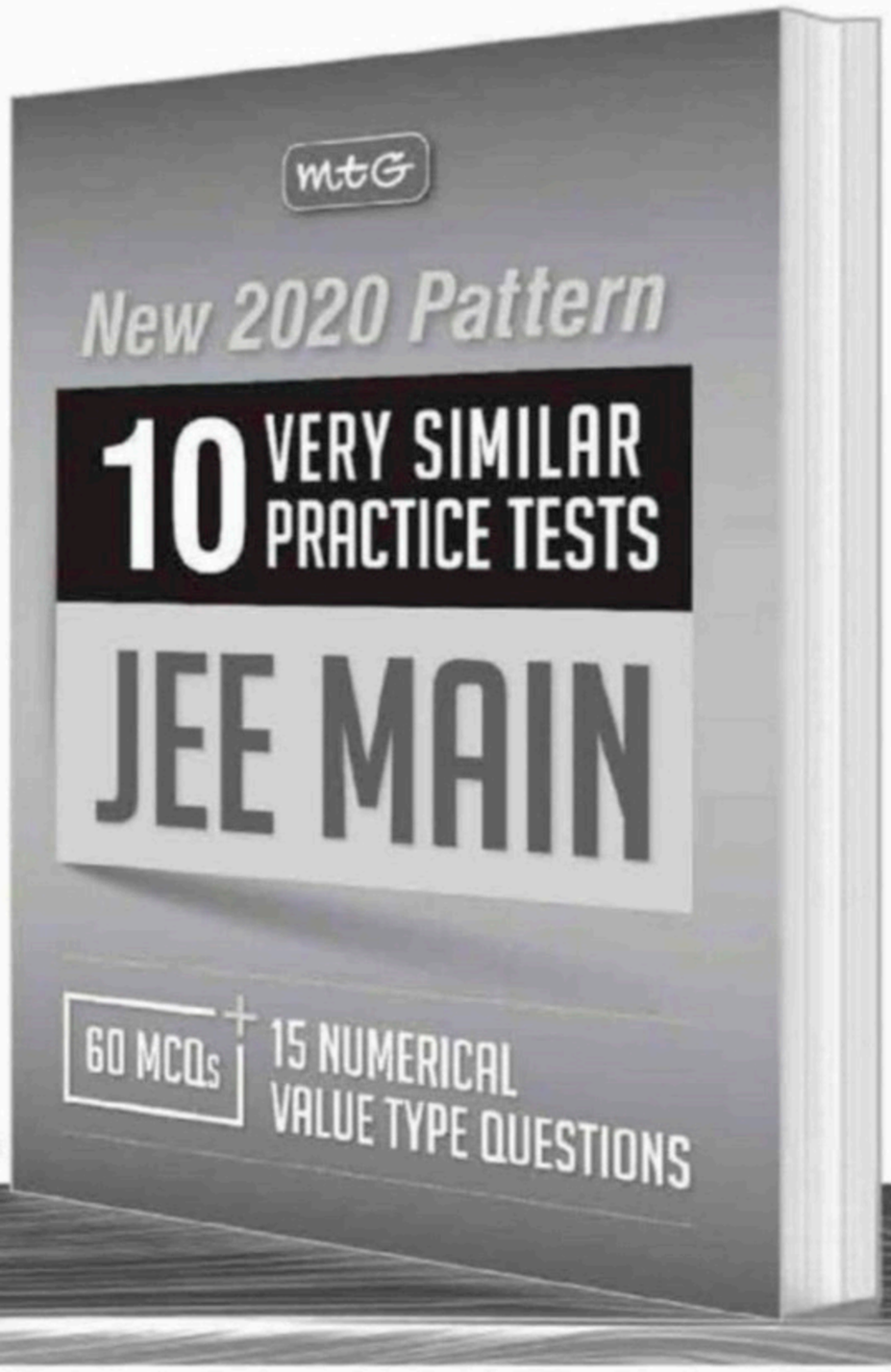

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$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2=(2\sin^{-1}x-A)^2$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2=4(\sin^{-1}x)^2-4A\sin^{-1}x+A^2$$

$$=4(y-A\cos^{-1}x-B)-4A\sin^{-1}x+A^2 \text{ (Using (i))}$$

$$=4y-4B-4A\cdot\frac{\pi}{2}+A^2 \quad \left(\because \cos^{-1}x=\frac{\pi}{2}-\sin^{-1}x\right)$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2=4y-2A\pi-4B+A^2 \qquad \text{... (ii)}$$

$$=4y+\text{constant}$$

Differentiating (ii) w.r.t. x, we get

$$2(1-x^2)\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=4\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=\lambda=2 \Rightarrow \lambda^2=4$$

22. (7) : From the given differential equation, we have

$\sin y(3\sin x-7\cos y+7)dy+\cos x(3\cos y-7\sin x-3)dx=0$

$\Rightarrow \sin y[(5-2)\sin x-(5+2)\cos y+(5+2)]dy$
$+\cos x[(5-2)\cos y-(5+2)\sin x-(5-2)]dx=0$

$\Rightarrow 5\sin y(\sin x-\cos y+1)dy+5\cos x(\cos y-\sin x-1)dx$
$-2\sin y(\sin x+\cos y-1)dy-2\cos x(\cos y+\sin x-1)dx=0$

$\Rightarrow 5(\sin x-\cos y+1)(\sin y dy-\cos x dx)$
$-2(\sin x+\cos y-1)(\sin y dy+\cos x dx)=0$

$$\Rightarrow \frac{5(\cos x dx-\sin y dy)}{\sin x+\cos y-1}+\frac{2(\cos x dx+\sin y dy)}{\sin x-\cos y+1}=0$$

$$\Rightarrow \frac{5d(\sin x+\cos y-1)}{\sin x+\cos y-1}+\frac{2d(\sin x-\cos y+1)}{\sin x-\cos y+1}=0$$

$\Rightarrow 5\log(\sin x+\cos y-1)+2\log(\sin x-\cos y+1)=\log c$

$\Rightarrow (\sin x+\cos y-1)^5\cdot(\sin x-\cos y+1)^2=c$

$\Rightarrow (\sin x+\cos y-1)^{\lambda_1}(\sin x-\cos y+1)^{\lambda_2}=c$

$\Rightarrow \lambda_1=5, \lambda_2=2 \Rightarrow \lambda_1+\lambda_2=5+2=7$

23. (26) : We know that if solution is of the form $y=(A+Bx+Cx^2)e^{-kx}$, then differential equation is given by $(D+k)^3y=0$

$$\text{i.e., } \frac{d^3y}{dx^3}+3k\frac{d^2y}{dx^2}+3k^2\frac{dy}{dx}+k^3y=0$$

Putting $k=2$, we have

$$y_3+6y_2+12y_1+8y=0=y_3+k_1y_2+k_2y_1+k_3y$$

$\therefore \quad k_1+k_2+k_3=6+12+8=26$

24. (2) : Given differential equation is

$$y^2(y-xp)=x^4p^2 \quad \left(\text{where } p=\frac{dy}{dx}\right) \qquad \text{... (i)}$$

Let $x=\frac{1}{X}$ and $y=\frac{1}{Y}$

$$\therefore\ dx=-\frac{1}{X^2}dX \text{ and } dy=-\frac{1}{Y^2}dY$$

Now, $\frac{dy}{dx}=p=\frac{X^2}{Y^2}\frac{dY}{dX}=\frac{X^2}{Y^2}P \quad \left(\text{where, } P=\frac{dY}{dX}\right)$

Now, from (i), we have $\frac{1}{Y^2}\left(\frac{1}{Y}-\frac{1}{X}\frac{X^2}{Y^2}P\right)=\frac{1}{X^4}\frac{X^4}{Y^4}P^2$

$\Rightarrow Y-XP=P^2$ or $Y=PX+P^2$ (Clairaut's form) whose solution is given by

$$Y=cX+c^2 \Rightarrow \frac{1}{y}=\frac{c}{x}+c^2$$

$$\Rightarrow \frac{1}{y}=\frac{c}{x}+c^\lambda \Rightarrow \lambda=2$$

25. (4) : Given differential equation is

$$y=2xp-y^2p^3 \quad \left(\text{where } p=\frac{dy}{dx}\right) \qquad \text{....(i)}$$

$$\Rightarrow 2xp=y+y^2p^3 \Rightarrow x=\frac{y}{2p}+\frac{1}{2}p^2y^2$$

Differentiating w.r.t. y, we have

$$\frac{dx}{dy}=\frac{1}{2p}\cdot 1+y\left(-\frac{1}{2p^2}\right)\frac{dp}{dy}+\frac{1}{2}\left[p^2(2y)\cdot 1+y^2\left(2p\frac{dp}{dy}\right)\right]$$

$$\Rightarrow \frac{1}{p}=\frac{1}{2p}-\frac{y}{2p^2}\frac{dp}{dy}+p^2y+y^2p\frac{dp}{dy}$$

$$\Rightarrow \left(\frac{1}{p}-\frac{1}{2p}\right)-p^2y=\left(y^2p-\frac{y}{2p^2}\right)\frac{dp}{dy}$$

$$\Rightarrow \left(\frac{1}{2p}-p^2y\right)=-\frac{y}{p}\left(\frac{1}{2p}-p^2y\right)\frac{dp}{dy}$$

$$\Rightarrow \frac{1}{2p}-p^2y=0 \text{ and } 1+\frac{y}{p}\frac{dp}{dy}=0$$

$$\left(\text{Neglect } \frac{1}{2p}-p^2y \text{ as it does not contain } \frac{dp}{dy}\right)$$

$$\therefore \quad 1+\frac{y}{p}\frac{dp}{dy}=0 \Rightarrow \frac{dy}{y}+\frac{dp}{p}=0$$

$$\Rightarrow \log(yp)=\log c \Rightarrow yp=c \Rightarrow p=\frac{c}{y} \qquad \text{...(ii)}$$

Now using (ii) in (i), we get

$$y=2x\cdot\frac{c}{y}-\frac{y^2c^3}{y^3} \Rightarrow y^2=2xc^1-c^3$$

$\Rightarrow y^2=2x\,c^m-c^n$

$\therefore \quad m+n=1+3=4$

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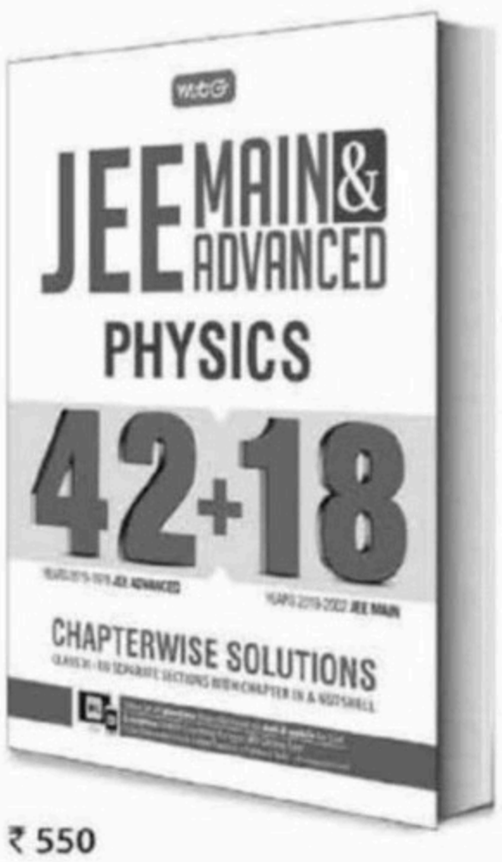

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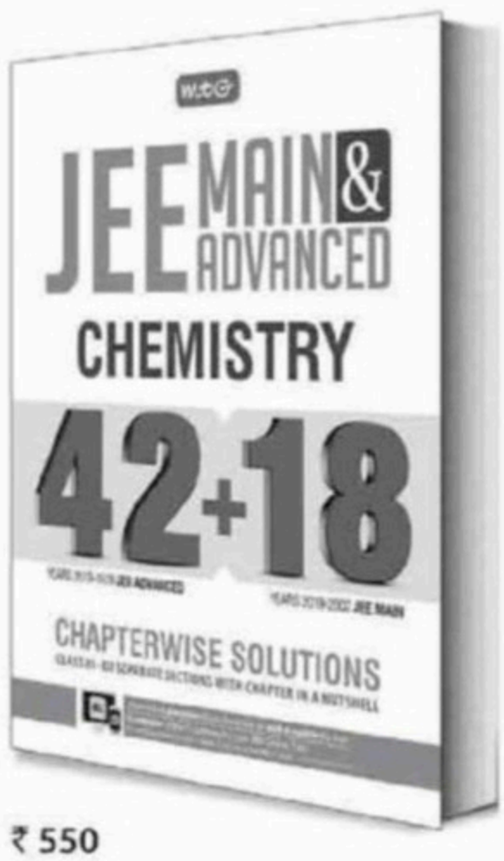

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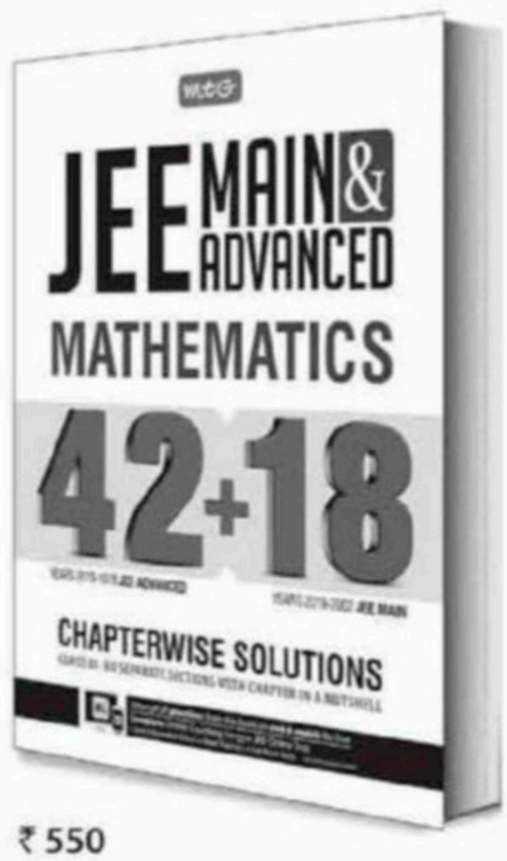

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MATHS MUSING

SOLUTION SET-207

1. (a) : $\sum_{i=0}^{n}\sum_{j=1}^{n} {}^nC_j\,{}^jC_i = {}^nC_1\,({}^1C_0 + {}^1C_1) +$

$${}^nC_2({}^2C_0 + {}^2C_1 + {}^2C_2) + {}^nC_3({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3)$$
$$+ {}^nC_4({}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4)$$
$$+\ldots+ {}^nC_n({}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n)$$
$$= {}^nC_1\,(2) + {}^nC_2\,(2)^2 + {}^nC_3\,(2)^3 + {}^nC_4\,(2)^4 + \ldots + {}^nC_n\,(2)^n$$
$$= (1+2)^n - 1 = 3^n - 1$$

2. (b) : As the given plane passes through $\vec{c}$ and is parallel to the vector $\vec{b}-\vec{c}$ and $\vec{a}$. So it is normal to $(\vec{b}-\vec{c})\times\vec{a}$. Hence its equation is given as

$$(\vec{r}-\vec{c})\cdot[(\vec{b}-\vec{c})\times\vec{a}]=0 \qquad \ldots(i)$$

$\therefore$ Length of perpendicular from origin $O(0, 0, 0)$ to (i) is given by

$$\frac{|\vec{0}\cdot(\vec{b}\times\vec{a}+\vec{a}\times\vec{c})-[\vec{c}\,\vec{b}\,\vec{a}]|}{|\vec{b}\times\vec{a}+\vec{a}\times\vec{c}|} = \frac{1}{|\vec{b}\times\vec{a}+\vec{a}\times\vec{c}|}[\vec{c}\,\vec{b}\,\vec{a}]$$

3. (a) : $f(1) = 0 \Rightarrow a + b + c = 0$

$f(7) \in (50, 60) \Rightarrow 50 < 49a + 7b + c < 60$

$\Rightarrow 50 < 48a + 6b < 60 \Rightarrow 8a+b \in \left(\frac{25}{3}, 10\right)$

$\Rightarrow 8a + b = 9$

Similarly, $9a + b = 11$

Hence, $a = 2, b = -7, c = 5$

$\Rightarrow f(x) = 2x^2 - 7x + 5 \Rightarrow f(2) = 8 - 14 + 5 = -1$

4. (c) : Given pair of lines are

$(\sqrt{3}x - y)(x - \sqrt{3}y) = 0$ or $y = \tan 60°x, y = \tan 30°x$

After rotation, the separate equations are

$y = \tan 90°x, y = \tan 60°x \Rightarrow x = 0, y = \sqrt{3}x$

$\therefore$ Combined equation is $x(\sqrt{3}x - y) = 0$

or $\sqrt{3}x^2 - xy = 0$

5. (a) : We have, $a\sin^2 x + b\cos^2 x = c$

$\Rightarrow a\tan^2 x + b = c(1 + \tan^2 x)$

$\Rightarrow \tan^2 x = \left(\frac{c-b}{a-c}\right)$

Similarly, $\tan^2 y = \left(\frac{d-a}{b-d}\right)$

$\therefore \quad \frac{a^2}{b^2} = \frac{\tan^2 y}{\tan^2 x} = \frac{(a-d)(c-a)}{(b-c)(d-b)}$

6. (c, d) : We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

But it is given that $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$

$\therefore\ P(A \cap B) = P(A)\ P(B)$ *i.e.*, A, B are independent events.

$\therefore\ A, \bar{B}$; $\bar{A}, B$ and $\bar{A}, \bar{B}$ are also pairwise independent events.

$\therefore\ P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$

Now, $P(A\,|\,B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$

7. (c) : $z_1 = -\frac{(-4+3i)(4+3i)}{3+4i} = 3 - 4i$

8. (b) : Circumcentre of $\Delta ABC = (0, 0)$

Now, centroid of $\Delta ABC = \frac{3+10i}{3}$

$\therefore$ Orthocentre of $\Delta ABC = 3 + 10i$.

9. (100) : The last digit of any power of 3 can be 3, 9, 7, 1. Similarly last digit of any power of 7 can be 7, 9, 3, 1

Total number of ways $= 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5$
$= 100$

10. (d) : P $\to$ 2; Q $\to$ 3; R $\to$ 1

(P) $xy = c^2 = -$ve ($\because$ c is purely imaginary)

$\Rightarrow$ If x is positive y must be negative real $\Rightarrow \alpha = \frac{1-\sqrt{3}}{2}$

(Q) We have, $2b = a + c$ [$\because$ a, b, c are in A.P.]

Now, $(a+c-b)s = kb^2 \Rightarrow \frac{b(a+b+c)}{2} = kb^2$

$\Rightarrow \frac{3b^2}{2} = kb^2 \Rightarrow k = \frac{3}{2}$

(R) $r = \sqrt{(3-0)^2 + (4-0)^2} - 2 = 3$

COMIC CAPSULE

JEE Main

NUMERICAL VALUE TYPE QUESTIONS

Exam on
5th, 7th to 9th
&
11th April, 2020

1. The eccentricity of the ellipse $25x^2 + 16y^2 - 150x - 175 = 0$ is _____.

2. For a sample of size 60, we have the information $\sum x_i^2 = 18000$ and $\sum x_i = 960$, then the variance is _____.

3. If $a = 9, b = 8$ and $c = x$ satisfies $3\cos C = 2$, then the value of x is _____.

4. Let $f(x) = \max\{2 - x, 2, 1 + x\}$. Then $\int_{-1}^{1} f(x)\,dx =$ _____.

5. Numbers greater than 1000 but less than 4000 are formed using the digits 0, 2, 3, 4 when repetition allowed. Then number of numbers formed is _____.

6. If the mean and variance of a binomial variate x are 2 and 1 respectively and the probability that x takes the value greater than or equal to 1 is λ, then value of $\frac{16\lambda}{5}$ is _____.

7. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is _____.

8. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is _____.

9. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals _____.

10. If $A = \begin{bmatrix} 5 & -6 \\ 1 & -1 \end{bmatrix}$ then determinant of $(A^{1003} - 5A^{1002})$ is _____.

11. If x, y, z are distinct positive numbers such that $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$, then the value of xyz is _____.

12. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. If $\vec{a} = \lambda(\vec{b} \times \vec{c})$, then value of $|\lambda|$ is _____.

13. Let $f(x)$ be the ratio of two quadratic polynomials. If $f(0) = 6$ and $f(x)$ assumes turning values 3 and 4 at $x = 2$ and $x = -2$ respectively, then $f(1) =$ _____.

14. If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$, then k is _____.

15. If $\frac{dy}{dx} + y\tan x = \sec x$, $y(0) = 0$, then $y\left(\frac{\pi}{2}\right) =$ _____.

SOLUTIONS

1. (0.6) : $25(x - 3)^2 + 16y^2 = 400$

$$\frac{(x-3)^2}{16} + \frac{y^2}{25} = 1 \quad \therefore \quad e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5} = 0.6$$

2. (44) : Since, $\sigma^2 = \frac{\sum x_i^2}{60} - \left(\frac{\sum x_i}{60}\right)^2$

$$= \frac{18000}{60} - \left(\frac{960}{60}\right)^2 = 300 - 256 = 44$$

3. (7) : $\cos C = \frac{2}{3} = \frac{81 + 64 - x^2}{2 \cdot 9 \cdot 8} \Rightarrow x^2 = 49 \Rightarrow x = 7$

4. (4.5) : In $(-1, 0)$, $2 - x$ is maximum and in $(0, 1)$, 2 is maximum.

$$\therefore \quad \int_{-1}^{1} f(x)dx = \int_{-1}^{0} (2 - x)dx + \int_{0}^{1} 2\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_{-1}^{0} + 2 = \frac{5}{2} + 2 = \frac{9}{2} = 4.5$$

5. (128) : Let number of numbers formed be x.

$\therefore$ $1000 < x < 4000$, which means left extreme digit will be either 2 or 3.

$\therefore$ Required numbers $= {}^2C_1 \times$ H T U

[where H = Hundred's place, T = Ten's place and U = Unit's place]

$= {}^2C_1 \times 4 \times 4 \times 4 = 128$

6. (3) : Given, mean of binomial distribution $= np = 2$ and variance $= npq = 1$

$\Rightarrow \quad p = q = \frac{1}{2}$ and $n = 4$

Now, $\lambda = P(x \geq 1) = P(1) + P(2) + P(3) + P(4)$

$$= 1 - P(0) = 1 - {}^4C_0\left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16}$$

$\Rightarrow \quad \lambda = \frac{15}{16} \quad \therefore \quad \frac{16\lambda}{5} = 3$

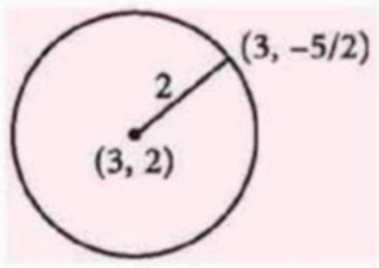
7. (3.5) : The planes are

$4x + 2y + 4z = 16, \quad 4x + 2y + 4z = -5$

$$\text{Distance between planes} = \frac{16 - (-5)}{\sqrt{4^2 + 2^2 + 4^2}} = \frac{21}{6} = 3.5$$

8. (5) : $|2z - 6 + 5i|$

$$= 2\left|z - \left(3 - \frac{5}{2}i\right)\right|$$



$$\text{Min. distance} = \left(2 + \frac{5}{2}\right) - 2 = \frac{5}{2}$$

$$\text{Hence, } 2\left|z - \left(3 - \frac{5}{2}i\right)\right|_{\min.} = 2 \cdot \frac{5}{2} = 5$$

9. (7) : $T_n = {}^nC_3$

$\therefore \quad T_{n+1} - T_n = 21 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow \quad {}^nC_2 + {}^nC_3 - {}^nC_3 = 21 \Rightarrow {}^nC_2 = 21$

$\Rightarrow \quad \frac{1}{2}n(n-1) = 21 \Rightarrow n^2 - n - 42 = 0$

$\Rightarrow \quad (n - 7)(n + 6) = 0$

As $n \geq 3$, we get $n = 7$

10. (6) : $|A^{1003} - 5A^{1002}| = |A^{1002}(A - 5I)|$

$$= |A^{1002}|\,|A - 5I| = |A|^{1002}\,|A - 5I| = 1 \times \begin{vmatrix} 0 & -6 \\ 1 & -6 \end{vmatrix} = 6$$

11. (1) : $x - y = \frac{y - z}{yz}, y - z = \frac{z - x}{zx}, z - x = \frac{x - y}{xy}$

Multiplying above all, we get $xyz = 1$ (As $xyz > 0$)

12. (2) : Given $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

$\vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{c}$.

$\therefore \quad \vec{a}$ is perpendicular to the plane of $\vec{b}$ and $\vec{c}$.

Also $\vec{a}$ is a unit vector.

$$\therefore \quad \vec{a} = \pm\frac{\vec{b} \times \vec{c}}{|\vec{b} \times \vec{c}|} \qquad \text{...(i)}$$

But $|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}|\sin\frac{\pi}{6} = 1 \cdot 1 \cdot \frac{1}{2}$

$\therefore$ From (i), we have $\vec{a} = \pm 2(\vec{b} \times \vec{c}) \quad \therefore \quad |\lambda| = 2$

13. (2.8) : $f(x) = \frac{A(x)}{B(x)}$, $A(x) = 6(ax^2 + bx + 1)$,

$B(x) = px^2 + qx + 1$

Now, $f(2) = 3, f(-2) = 4$

$\Rightarrow \quad A(2) = 3B(2), A(-2) = 4B(-2)$ ...(i)

$f'(2) = 0, f'(-2) = 0 \Rightarrow A'(2) = 3B'(2)$

and $A'(-2) = 4B'(-2)$ ...(ii)

(i) and (ii) $\Rightarrow 2(4a + 2b + 1) = 4p + 2q + 1$,

$3(4a - 2b + 1) = 2(4p - 2q + 1)$,

$2(4a + b) = 4p + q$, and $3(-4a + b) = 2(-4p + q)$

On solving, we get $a = \frac{1}{4}, b = -3, p = \frac{1}{4}, q = -5$

$$\therefore \quad f(x) = \frac{6(x^2 - 12x + 4)}{(x^2 - 20x + 4)} \text{ and } f(1) = \frac{14}{5} = 2.8$$

14. (7) : The point $(4, 2, k)$ lies on the plane.

$\therefore \quad 8 - 8 + k = 7 \Rightarrow k = 7$

15. (1) : We have a linear D.E. with I.F. $= \exp\int \tan x\,dx$ $= \sec x$

$\therefore$ Solution is $y \sec x = \int \sec^2 x\,dx = \tan x + c$

When $x = 0, y = 0 \Rightarrow c = 0 \quad \therefore \quad y = \sin x, \; y\left(\frac{\pi}{2}\right) = 1$

SAMURAI SUDOKU

ANSWER · MARCH 2020

9	5	4	8	1	3	7	2	6
6	1	7	4	2	9	3	5	8
2	8	3	6	5	7	4	9	1
1	3	8	9	7	5	6	4	2
4	6	9	2	8	1	5	7	3
5	7	2	3	4	6	8	1	9
3	4	6	7	9	2	1	8	5
7	9	1	5	6	8	2	3	4
8	2	5	1	3	4	9	6	7

5	7	6	2	8	9	4	3	1
1	2	3	6	7	4	9	5	8
9	4	8	1	3	5	6	2	7
4	8	2	5	9	3	1	7	6
3	5	7	8	1	6	2	4	9
6	1	9	4	2	7	5	8	3
7	6	4	3	5	1	8	9	2
8	9	1	7	4	2	3	6	5
2	3	5	9	6	8	7	1	4

3	4	6	7	9	2	1	8	5
7	9	1	5	6	8	2	3	4
8	2	5	1	3	4	9	6	7

3	2	9	7	6	4	3	5	1
7	5	6	8	9	1	7	4	2
4	1	8	2	3	5	9	6	8

7	2	9	1	6	3	5	4	8
6	5	3	2	8	4	1	7	9
4	1	8	9	7	5	6	2	3

7	5	3	2	9	6	8	4	1
4	2	1	7	8	3	5	9	6
8	9	6	4	5	1	3	7	2

6	3	7	9	5	2	8	4	1
8	4	2	3	1	7	5	4	2
5	9	1	4	8	6	1	7	9

3	6	4	1	7	9	2	5	8
9	6	8	7	5	4	2	9	6
3	2	5	8	4	1	7	6	5

9	1	4	6	3	2	7	5	8
6	3	8	1	7	5	4	2	9
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2	8	7	3	1	4	9	6	5
3	6	5	9	2	7	1	8	4
1	4	9	5	6	8	2	3	7

1	4	9	6	2	5	7	8	3
2	3	5	9	8	7	1	4	6
6	7	8	3	1	4	2	5	9
5	6	1	4	9	3	8	7	2
7	9	4	2	5	8	6	3	1
8	2	3	7	6	1	5	9	4

JEE 2020 ADVANCED

PRACTICE PAPER

Exam on 17th May

* ALOK KUMAR

Single Option Correct Type

1. The point of intersection of common tangents of $y^2 = 4ax (a > 0)$ and $x^2 + y^2 - 6ax + a^2 = 0$ is

(a) $(a, 0)$ (b) $(-a, 0)$

(c) $(-a, -a)$ (d) $(-2a, 0)$

2. A variable straight line with slope $m (m \neq 0)$ intersects the hyperbola $xy = 1$ at two distinct points. Then the locus of the point which divides the line segment between these two points in the ratio 1 : 2 is a/an

(a) ellipse (b) hyperbola

(c) circle (d) parabola

3. If $\int (2-3\sin^2 x)\sqrt{\sec x}\, dx = 2f(x)\sqrt{g(x)} + c$ and $f(x)$ is non constant function, then

(a) $f^2(x) + g^2(x) = 1$ (b) $f^2(x) - g^2(x) = 1$

(c) $f(x)g(x) = 1$ (d) $f(x) = g(x)$

4. Three distinct vertices are randomly chosen among the vertices of a cube. The probability that they are vertices of an equilateral triangle is

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$ (c) $\frac{1}{7}$ (d) $\frac{6}{7}$

5. The numbers a, b are chosen from the set $\{1, 2, 3, 4, ..., 10\}$ such that $a \leq b$ with replacement. The probability that a divides b is

(a) $\frac{5}{11}$ (b) $\frac{29}{55}$

(c) $\frac{27}{55}$ (d) none of these

6. For $\lambda \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the equation $\cos x + \sin(\lambda + x) - \sin(\lambda - x) = 2$ has atleast one solution if $\lambda \in$

(a) $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ (b) $\left[-\frac{\pi}{3}, \frac{\pi}{6}\right]$

(c) $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$ (d) $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

7. The greatest and the least value of $|z_1 + z_2|$ if $z_1 = 24 + 7i$ and $|z_2| = 6$ are respectively

(a) 31, 19 (b) 25, 19

(c) 31, 25 (d) none of these

8. Let α, β be the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$, then $A_{n+1} - aA_n + bA_{n-1}$ is equal to

(a) $-a$ (b) b (c) 0 (d) $a - b$

9. The number of permutations of the set $\{1, 2, 3, 4\}$ in which no two adjacent positions are filled by consecutive integers (increasing order)

(a) is an even integer

(b) is a composite number divisible by 3

(c) does not exceed 17

(d) is at most 19

10. The inverse of a symmetric matrix (if it exists) is

(a) a symmetric matrix

(b) a skew symmetric matrix

(c) a diagonal matrix

(d) none of these

11. If $I = \int_0^a \sqrt{\frac{a-x}{a+x}}\, dx$, $a > 0$ then I equals

(a) $\frac{1}{2}\left(a - \frac{\pi}{2}\right)$ (b) $\frac{a}{2}(\pi - 1)$

(c) $\frac{1}{\sqrt{2}}a(\pi - 1)$ (d) $a\left(\frac{\pi}{2} - 1\right)$

12. If $I = \int (\sqrt{\tan x} + \sqrt{\cot x})dx$, then I equals

(a) $\sqrt{2}\sin^{-1}(\sin x + \cos x) + C$

(b) $\sqrt{2}\cos^{-1}(\sin x - \cos x) + C$

(c) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + C$

(d) $\sqrt{2}\cos^{-1}(\sin x + \cos x) + C$

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13. Let $f(x) = (1+x)^n - (1+nx)$, $x \in [-1, \infty)$. Then f
(a) has an absolute maximum at $x = 0$.
(b) has neither absolute maximum nor absolute minimum at $x = 0$.
(c) has an absolute minimum at $x = 0$.
(d) does not have absolute minimum at $x = 0$.

14. If $f(x) = (x/2) - 1$, then on the interval $[0, \pi]$
(a) $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both continuous.
(b) neither $\tan(f(x))$ nor $\dfrac{1}{f(x)}$ is continuous.
(c) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous.
(d) neither $\tan(f(x))$ nor $f^{-1}(x)$ is continuous.

15. Let the function $f(x) = x^2 + x + \sin x - \cos x + \log(1 + |x|)$ be defined on the interval $[0, 1]$. The function $g(x)$ on $[-1, 1]$ satisfying $g(-x) = -f(x)$ is
(a) $x^2 + x + \sin x + \cos x - \log(1 + |x|)$
(b) $-x^2 + x + \sin x + \cos x - \log(1 + |x|)$
(c) $-x^2 + x + \sin x - \cos x + \log(1 + |x|)$
(d) none of these

16. An equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$ with the vertex at $(a, 0)$. The equation of the side opposite to this vertex is
(a) $2x - a = 0$ (b) $x + a = 0$
(c) $2x + a = 0$ (d) $3x - 2a = 0$

17. Let 'k' be a real number. Number of real roots of the equation $(3x^2 + kx + 3)(x^2 + kx - 1) = 0$, is
(a) atleast two (b) atmost two
(c) exactly two (d) cannot be determined

18. If $\lim\limits_{x\to\infty} \dfrac{4x^2+3x+5}{x+1+x^{k-1}}$ exists, then
(a) $k = 2$ (b) $k < 2$ (c) $k \geq 3$ (d) $k < -3$

19. The value of $\sum\limits_{k=1}^{\infty}\sum\limits_{n=1}^{\infty} \dfrac{k}{2^{n+k}}$ equals
(a) 5 (b) 4 (c) 3 (d) 2

20. The value of $\sum\limits_{r=1}^{20} r \cdot r!$ equals
(a) (10)! (b) (11)!
(c) (21)! –1 (d) (11)! – 1

One or More Than One Option(s) Correct Type

21. If $a \neq 0$ is a real number, then for the function $y = \dfrac{x+a}{\sqrt{x^2+a^2}}$
(a) $y \leq \sqrt{2}$ (b) $y > -1$
(c) $y < 1$ (d) $y \in (-1, \sqrt{2}]$

22. If A and B are two independent events such that $P(A) = 1/2$ and $P(B) = 1/5$, then
(a) $P(A \cup B) = 3/5$ (b) $P(A|B) = 1/2$
(c) $P(A|(A \cup B)) = 5/6$ (d) $P((A \cap B)|(A' \cup B')) = 0$

23. If $0 < x, y \leq 180°$ and $\sin(x - y) = \cos(x + y) = 1/2$, then the values of x and y are given by
(a) $x = 45°, y = 15°$ (b) $x = 45°, y = 135°$
(c) $x = 165°, y = 15°$ (d) $x = 165°, y = 135°$

24. A solution of the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$, where θ lies in the interval $(-\pi/2, \pi/2)$ is given by
(a) $\theta = 0$ (b) $\theta = \pi/3$
(c) $\theta = -\pi/3$ (d) $\theta = \pi/6$

25. If θ is eliminated from the equation $a\sec\theta - x\tan\theta = y$ and $b\sec\theta + y\tan\theta = x$ (a and b are constants), then the eliminated denotes the equation of
(a) the director circle of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.
(b) a circle of radius $(a^2 + b^2)^{1/2}$.
(c) director circle of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.
(d) director circle of the circle $x^2 + y^2 = \dfrac{a^2+b^2}{2}$.

26. If a and b are distinct positive integers and the quadratic equations $(a-1)x^2 - (a^2+2)x + (a^2+2a) = 0$ and $(b-1)x^2 - (b^2+2)x + (b^2+2b) = 0$ have a common root. Then which of the following can be true?
(a) $a^2 + b^2 = 45$ (b) $a = 2b$
(c) $b = 2a$ (d) $ab = 18$

27. $f: R \to R, f(x^2+x+3) + 2f(x^2-3x+5) = 6x^2 - 10x + 17\ \forall\ x \in R$, then
(a) $f(x)$ is strictly increasing
(b) $f(x) = 0$ has a root in $(0, 2)$
(c) $f(x)$ is an odd function (d) $f(x)$ is invertible

28. The domain of definition of the function, $f(x) = [2\tan\pi x]^{\log[2\tan\pi x]}\left(\dfrac{x^2+2x-3}{4x-4x-3}\right)$, where $[\cdot]$ denotes the greatest integer function is given by the interval $\left[n + \dfrac{1}{4}, n + \dfrac{1}{2}\right)$ where $n \in I$ then n can be equal to
(a) –5 (b) 2 (c) 0 (d) 3

Comprehension Type

Paragraph for Q.No. 29 to 31

Let m, n be two positive real numbers and define $f(n) = \int_0^{\infty} x^{n-1}e^{-x}dx$ and $g(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$.
It is known that $f(n)$, for $n > 0$ is finite and $g(m, n) = g(n, m)$ for $m, n > 0$.

29. $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx =$

(a) $g(m, n)$ (b) $g(m-1, n)$
(c) $g(m-1, n-1)$ (d) $g(m, n+1)$

30. $\int_0^1 x^m \left(\log_e \frac{1}{x}\right)^n dx =$

(a) $\frac{f(n+1)}{(m+1)^n}$ (b) $\frac{f(n)}{(m+1)^{(n+1)}}$

(c) $\frac{f(n+1)}{(m+1)^{(n+1)}}$ (d) $g(m+1, n+1)$

31. $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx =$

(a) $g(n, m)$ (b) $g(m-1, n+1)$
(c) $g(m-1, n-1)$ (d) $g(m+1, n-1)$

Paragraph for Q.No. 32 to 34

$L_1 : \frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

$L_2 : \frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

32. The lines L_1 and L_2 are

(a) Perpendicular (b) Parallel
(c) Coplanar (d) None of these

33. The lines L_1 and L_2 intersect at the point

(a) $(-3, 2, 1)$ (b) $(2, 1, -3)$
(c) $(1, -3, 2)$ (d) None of these

34. Equation of a plane containing L_1 and L_2 is

(a) $x+y+z=0$ (b) $3x-2y-z=0$
(c) $x-3y+2z=0$ (d) $x+y+z=42$

Paragraph for Q.No. 35 and 36

If $f''(x) < 0$ (> 0) on an interval (a, b) then the curve $y = f(x)$ on this interval is convex (concave), *i.e.* it is situated below (above) any of its tangent lines. If $f''(x_0) = 0$ or does not exist but $f'(x_0)$ exists and the second derivative changes sign when passing through the point x_0 then the point $(x_0, f(x_0))$ is the point of inflection of the curve $y = f(x)$.

35. If $y = x^4 + x^3 - 18x^2 + 24x - 12$, then

(a) $(-2, -24)$ is a point of inflection.
(b) $(-2, 3/2)$ is a point of inflection.
(c) $\left(3/2, -8\frac{1}{16}\right)$ is a point of inflection.
(d) y is convex on $(3/2, \infty)$.

36. If $y = x\sin(\log x)$, then

(a) y has only two points of inflection.
(b) y has only four points of inflection.
(c) $\pi/4$ is the only point of inflection.
(d) y has infinite number of points of inflection.

Matrix-Match Type

37. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}; \vec{b} = \hat{i}-\hat{j}+\hat{k}; \vec{c} = \hat{i}+\hat{j}-\hat{k}$, then match the following columns :

Column-I		Column-II	
(A)	$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} =$	(p)	8
(B)	$((\vec{a}\cdot\vec{c})\vec{c}+(\vec{c}\cdot\vec{b})\vec{a})\cdot\vec{b} =$	(q)	12
(C)	$(\vec{a}+2\vec{b})\cdot(\vec{a}+(\vec{a}\cdot\vec{c})\vec{b}) =$	(r)	–2
(D)	$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}+\vec{c}) =$	(s)	1

38. Match the following columns :

Column-I		Column-II	
(A)	$f(x) = x\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$, when $x \neq 0$ and $f(x) = 0$, when $x = 0$, then $f(x)$ is	(p)	$y_2 - m^2y = 0$
(B)	If $y = ae^{mx} + be^{-mx}$, then differential equation is	(q)	continuous at $x = 0$ but not differentiable
(C)	$y = x + \cos x + x^3 + 2x^2 + 6x + 17$ is	(r)	always decreasing
(D)	$y = \tan^{-1}x - x$ is	(s)	always increasing

SOLUTIONS

1. (b) : Solving $y^2 = 4ax$ and $x^2 + y^2 - 6ax + a^2 = 0$, we get points of contact as $(a, 2a)$, $(a, -2a)$.
Hence tangents intersect at $(-a, 0)$.

2. (b) : Let the points of intersection be $(t_1, 1/t_1)$, $(t_2, 1/t_2)$.

Now, $m = -\frac{1}{t_1t_2}$ or $t_1t_2 = -\frac{1}{m}$

Also by section formula, we have,

$(x, y) = \left(\frac{2t_1+t_2}{3}, \frac{2t_2+t_1}{3t_1t_2}\right)$

Solving for t_1, t_2 and eliminating them gives $2m^2x^2 + 5mxy + 2y^2 = m$ which is always a hyperbola as

$\frac{25m^2}{4} - 4m^2 = \frac{9m^2}{4} > 0, \forall m \neq 0$

3. (a) : $\int \frac{2-3\sin^2 x}{\sqrt{\cos x}}dx = \int \frac{2\cos^2 x - \sin^2 x}{\sqrt{\cos x}}dx$

$= 2\int (\cos x)\sqrt{\cos x}\,dx - \int \frac{\sin^2 x}{\sqrt{\cos x}}dx = 2\sin x\sqrt{\cos x} + c$

$\Rightarrow$ $f(x) = \sin x$ and $g(x) = \cos x$

4. (c) : The 3 vertices can be chosen among 8 in ${}^8C_3 = 56$ ways. For an equilateral triangle to be formed, sides must be chosen along face diagonals .Since through each vertex, 3 face diagonals can be drawn, so totally $\frac{8\times 3}{3} = 8$ ($\because$ Each equilateral triangle is counted 3 times here) such triangles can be formed.

$\therefore$ Required probability $= \frac{8}{56} = \frac{1}{7}$

5. (c) : Number of ways of choosing a and b such that $a \le b$ are $10 + 9 + 8 + \ldots + 1 = 55$

Number of ways of choosing a and b such that a divides b are $10 + 5 + 3 + 2 + 2 + 5 = 27$

$\therefore$ Required probability= 27/55

6. (d) : The given equation $\cos x + 2\cos\lambda \sin x = 2$ has atleast one solution if

$2 \le \sqrt{1+4\cos^2\lambda} \Rightarrow |\cos\lambda| \ge \frac{\sqrt{3}}{2} \Rightarrow \cos\lambda \ge \frac{\sqrt{3}}{2}$

as $\cos\lambda \ge 0$. Hence $\lambda \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

7. (a) : $|z_1 + z_2|$

$= |z_2 - (-24 - 7i)|$

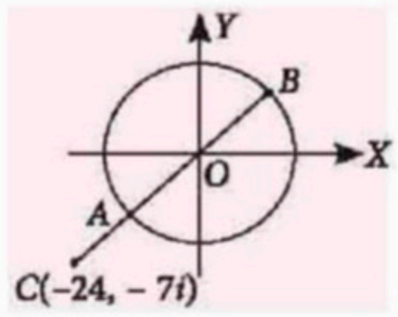
$|z_2| = 6$, $C(-24, -7i)$

$OC = 25$, $CA = OC - OA$

$= 25 - 6 = 19$

and $CB = OC + OB$

$= 25 + 6 = 31$



8. (c): $aA_n - bA_{n-1} = (\alpha+\beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$

$= \alpha^{n+1} + \beta^{n+1} + \alpha\beta^n + \beta\alpha^n - \alpha^n\beta - \alpha\beta^n = A_{n+1}$

9. (d) : Required number of ways

$= 4! - 3(3!) + 2(3!) + 2! - 1! = 19$

10. (a) : $AA^{-1} = A^{-1}A = I_n$

$\Rightarrow$ $(AA^{-1})' = (A^{-1}A)' = (I_n)'$

$\Rightarrow$ $(A^{-1})'A' = A'(A^{-1})' = I_n$

11. (d) : Consider $I = \int_0^a \frac{a-x}{\sqrt{a^2-x^2}}dx$

$= \left[a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2-x^2}\right]_0^a = a\left(\frac{\pi}{2}\right) + 0 - 0 - a = a\left(\frac{\pi}{2} - 1\right)$

12. (c) : Consider $I = \int \frac{\sin x + \cos x}{\sqrt{\cos x \sin x}}dx$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)\,dx = dt$

$\therefore$ $I = \int \frac{\sqrt{2}\,dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1}t + C$

13. (c) : $f(x) = (1+x)^n - (1+nx)$

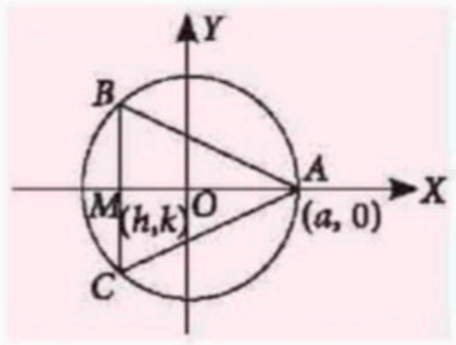
$\Rightarrow$ $f'(x) = n[(1+x)^{n-1} - 1] = 0$

$\Rightarrow$ $x = 0$. Also, $f(0) = 0$, $f(-1) = n - 1$

14. (c) : $f(x) = (x/2) - 1$ is continuous on $[0, \pi]$ and $y = \tan x$ is continuous on $[-1, \pi/2 - 1]$ = range of f

15. (b) : The function $g(x)$ is $-x^2 + x + \sin x + \cos x - \log(1 + |x|)$

16. (c) : Given ΔABC is inscribed in the circle $x^2 + y^2 = a^2$.



$\therefore$ $\frac{2h+a}{3} = 0$ and $\frac{2k+0}{3} = 0$

$\Rightarrow$ $h = -(a/2)$ and $k = 0$

17. (a) : D_1 for $3x^2 + kx + 3$ is $k^2 - 3b$ and D_2 for $x^2 + kx - 1$ is $k^2 + 4$. From D_1 and D_2 there exist at least two real roots of $(3x^2 + kx + 3)(x^2 + kx - 1) = 0$

18. (c) : $\lim_{x\to\infty} \frac{4x^2+3x+5}{x+1+x^{k-1}}$ will exist when power of x in denominator will be greater.

i.e., $k - 1 \ge 2 \Rightarrow k \ge 3$.

19. (d) : $\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}} = \sum_{k=1}^{\infty} k\left(\frac{1}{2^{k+1}} + \frac{1}{2^{k+2}} + \frac{1}{2^{k+3}} + \ldots\ldots\right)$

$= \sum_{k=1}^{\infty}\frac{k}{2^k} = 2$

20. (c) : $\sum_{r=1}^{20} r\cdot r! = \sum_{r=1}^{20}(r+1-1)\cdot r!$

$= \sum_{r=1}^{20}[(r+1)! - r!] = (21)! - 1$

21. (a, b, d) : $y = \frac{x+a}{\sqrt{x^2+a^2}}$ has global maximum value

$= \sqrt{2}$ at $x = a$ and

$\lim_{x\to\infty}\frac{x+a}{\sqrt{x^2+a^2}} = 1$, $\lim_{x\to-\infty}\frac{x+a}{\sqrt{x^2+a^2}} = -1$

$\therefore$ Range of $y = (-1, \sqrt{2}]$

22. (a, b, c, d) : $P(A \cup B) = 1 - P(A' \cap B') = 1 - P(A')P(B')$

[$\because$ A and B are independent]

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right) = 1 - \frac{2}{5} = \frac{3}{5}$

23. (a, d) : $\sin(x - y) = 1/2 \Rightarrow x - y = 30°$ or $150°$ ...(i)

and $\cos(x + y) = 1/2 \Rightarrow x + y = 60°$ or $300°$ ...(ii)

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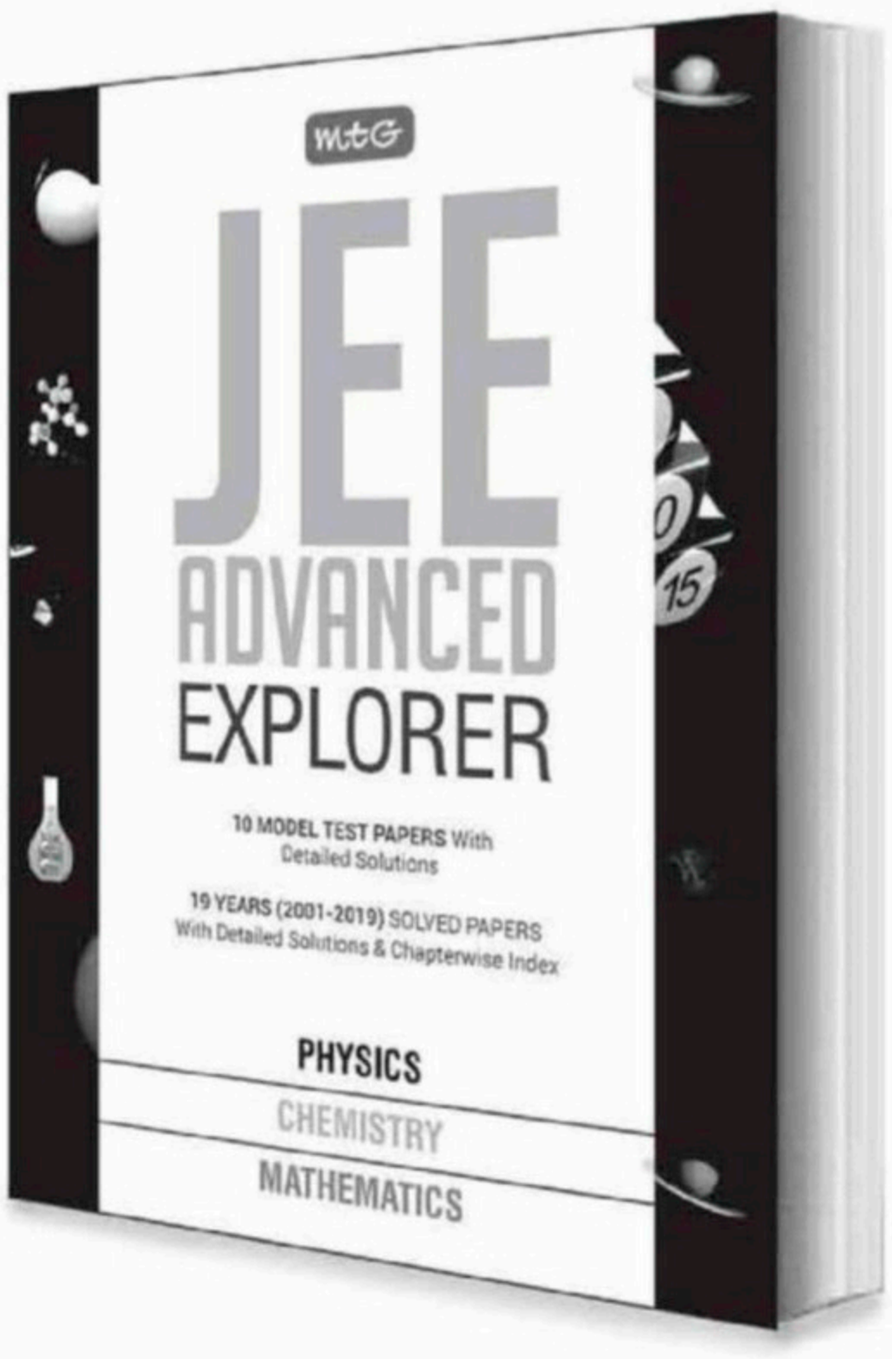

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From (i) and (ii), we get $x = 45°$, $y = 15°$ and $x = 165°$, $y = 135°$

24. (b, c) : We have, $(1 - \tan^2\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

$\Rightarrow \quad (1 - \tan^4\theta) + 2^{\tan^2\theta} = 0$

$\Rightarrow \quad 2^t = t^2 - 1$ (where $t = \tan^2\theta$)

25. (b, c, d) : Use $\sec^2\theta - \tan^2\theta = 1$

26. (b, c) : Use relation between roots and coefficients.

27. (a, b, d) : Let $f(x) = ax + b$ be a linear polynomial.

$\Rightarrow \quad f(x^2 + x + 3) + 2f(x^2 - 3x + 5) = 6x^2 - 10x + 17$

$[a(x^2 + x + 3) + b] + 2[a(x^2 - 3x + 5) + b] = 6x^2 - 10x + 17$

$\Rightarrow \quad a = 2, b = -3 \quad \therefore \quad f(x) = 2x - 3$

28. (a, b, c, d) : $f(x)$ is of the form $a^{\log_a N} = N$

$0 < [2\tan\pi x] \neq 1$ and $\dfrac{x^2+2x-3}{4x^2-4x-3} > 0$

$\Rightarrow \quad n + \dfrac{1}{4} \le x < n + \dfrac{1}{2} \quad \Rightarrow \quad n = 0, n \ge 2$ or $n \le -4$

29. (a) : In $g(m, n)$ putting $x = \dfrac{y}{1+y}$, $dx = \dfrac{1}{(1+y)^2}dy$, we get

$$g(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m-1}} \cdot \frac{1}{(1+y)^{n-1}} \cdot \frac{1}{(1+y)^2} dy$$

$$= \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

30. (c) : Put $\log_e \dfrac{1}{x} = t \Rightarrow x = e^{-t}$

So, $\displaystyle\int_0^1 x^m \left(\log_e \frac{1}{x}\right)^n dx = \int_0^\infty e^{-mt} t^n (e^{-t})\, dt$

$$= \int_0^\infty t^n e^{-(m+1)t} dt = \frac{1}{(m+1)^{(n+1)}} \int_0^\infty t^n e^{-t} dt = \frac{f(n+1)}{(m+1)^{(n+1)}}$$

31. (a) : Let $I = \displaystyle\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$

$$= \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx + \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx = I_1 + I_2$$

In I_2 put $x = \dfrac{1}{t}$ then $I_2 = \displaystyle\int_\infty^1 \frac{t^{n-1}}{\left(1+\frac{1}{t}\right)^{m+n}} \left(\frac{-dt}{t^2}\right)$

$$= \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

$$\therefore \quad I = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx + \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

$$= \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = g(m,n) = g(n,m)$$

32. (c) : We have, $\begin{vmatrix} -1-0 & 3-7 & -2+7 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$

$$= \begin{vmatrix} -1 & -4 & 5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = -(4+3) + 4(-6-1) + 5(9-2) = 0$$

So, L_1 and L_2 are coplanar.

33. (b) : Any point of L_1 is $(-3r_1 - 1, 2r_1 + 3, r_1 - 2)$ and on L_2 is $(r_2, -3r_2 + 7, 2r_2 - 7)$. Two points are same if $r_2 = 2$, $r_1 = -1$ and the point of intersection of L_1 and L_2 is $(2, 1, -3)$.

34. (a) : Plane containing L_1 and L_2 is given by

$$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad x + y + z = 0$

35. (c) : We have, $y = x^4 + x^3 - 18x^2 + 24x - 12$

$y' = 4x^3 + 3x^2 - 36x + 24$

$y'' = 12x^2 + 6x - 36 = 6(x + 2)(2x - 3)$

Hence $y'' = 0$ at $x_1 = -2$, $x_2 = 3/2$

Also, $y'' > 0$ on $(-\infty, -2)$ and $(3/2, \infty)$ and $y'' < 0$ on $(-2, 3/2)$.

36. (d) : We have, $y = x \sin(\log x)$

$\Rightarrow \quad y' = \sin(\log x) + \cos(\log x)$

$$\Rightarrow \quad y'' = \frac{1}{x}[\cos(\log x) - \sin(\log x)] = \frac{\sqrt{2}}{x}\sin\left(\frac{\pi}{4} - \log x\right)$$

Hence, $y'' = 0$ at $x_k = e^{\pi/4 + k\pi}$, $k = 0, \pm1, \pm2, \ldots\ldots$

37. (A) $\rightarrow$ (s) ; (B) $\rightarrow$ (r) ; (C) $\rightarrow$ (q) ; (D) $\rightarrow$ (p)

$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = 1 + (-1) + 1 = 1$

Similarly compute other expressions.

38. (A) $\rightarrow$ (q) ; (B) $\rightarrow$ (p) ; (C) $\rightarrow$ (s) ; (D) $\rightarrow$ (r)

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PRACTICE PROBLEMS

JEE

ADVANCED 2020

Exam Date
17th May 2020

SECTION-I

One or More Than One Option(s) Correct Type

1. For a positive integer n, let
$f_n(\theta) = \tan(\theta/2)(1+\sec\theta)(1+\sec 2\theta)...(1+\sec 2^n\theta)$, then
(a) $f_2(\pi/16) = 1$ (b) $f_3(\pi/32) = 1$
(c) $f_4(\pi/64) = 1$ (d) $f_5(\pi/128) = 1$

2. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then
(a) Equation of ellipse is $x^2 + 2y^2 = 2$
(b) The foci of ellipse are $(\pm 1, 0)$
(c) Equation of ellipse is $x^2 + 2y^2 = 4$
(d) The foci of ellipse are $(\pm\sqrt{2}, 0)$

3. If C_r stands for ${}^nC_r$, then the sum of the series
$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + ... + (-1)^n(n+1)C_n^2\right],$$
where n is an even positive integer, is equal to
(a) 0 (b) $(-1)^{n/2}(n+1)$
(c) $(-1)^n(n+2)$ (d) $(-1)^{n/2}(n+2)$

4. If the sequence $3, 5\left(1+\frac{1}{x}\right), 7\left(1+\frac{1}{x}\right)^2, 9\left(1+\frac{1}{x}\right)^3,$
$...., (2n+1)\left(1+\frac{1}{x}\right)^{n-1}$, ... till infinity $(n \in N)$, has finite sum 'S', then
(a) Range of x is $\left(-\infty, -\frac{1}{4}\right)$
(b) Range of x is $\left(-\infty, -\frac{1}{2}\right)$
(c) $S = x^2 - \frac{x}{2} + 1$ (d) $S = 2x^2 - x$

5. If domain of $f(x)$ is $(0, 1] \cup [e^2, e^3]$, then domain of $g(x) = 3 - f(2 + e^{3\{x\}})$, where $\{x\}$ denotes fractional part function of x, is
(a) $\bigcup_{n\in I}\left[n, n+\frac{2}{3}\right]$
(b) $\left[\frac{1}{3}\ln(e^2-2), \frac{1}{3}\ln(e^3-2)\right]$
(c) $\bigcup_{n\in I}\left[n+\frac{1}{3}\ln(e^2-2), n+\frac{1}{3}\ln(e^3-2)\right]$
(d) $\bigcup_{n\in I}\left[n, n+\frac{1}{3}\ln(e^3-2)\right]$

6. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq O$, when $n =$
(a) 57 (b) 55 (c) 58 (d) 56

7. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is
(a) 55 (b) 66 (c) 77 (d) 88

8. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are
(a) $\left(\frac{4}{3}, 3\right)$ (b) $\left(3, \frac{2}{3}\right)$ (c) $\left(3, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

9. If the sum of the first ten terms of the series
$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...,$ is $\frac{16}{5}m$, then m is equal to
(a) 102 (b) 101
(c) 100 (d) 99

10. $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\pi/2$ if x is equal to

(a) $-1/12$ (b) $1/6$ (c) $1/12$ (d) $-1/6$

11. The solution of $y_1(x^2y^3 + xy) = 1$ is

(a) $1/x = 2 - y^2 + Ce^{-y^2/2}$

(b) the solution of an equation which is reducible to linear equation.

(c) $2/x = 1 - y^2 + e^{-y/2}$

(d) $e^{y^2/2}\left(\frac{1-2x}{x} + y^2\right) = C$

12. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b}$ be two vectors perpendicular to each other in xy-plane. The vectors $\vec{c}$ in the same plane having projections 1 and 2 along $\vec{a}$ and $\vec{b}$ are

(a) $-\frac{2}{3}\hat{i} + \frac{11}{2}\hat{j}$ (b) $2\hat{i} - \hat{j}$

(c) $-\frac{2}{5}\hat{i} + \frac{11}{5}\hat{j}$ (d) $\frac{2}{3}\hat{i} + \frac{11}{2}\hat{j}$

13. The coordinates of the vertices of a tetrahedron $ABCD$ are as follows $A(2, 3, 4)$; $B(1, -1, 2)$; $C(0, 4, 5)$; $D(-2, 3, -4)$. Then which of the following is true?

(a) The angle between lines AB and CD is $\cos^{-1}\left(\frac{24}{\sqrt{1860}}\right)$

(b) The angle between AD and the plane ABC is $\sin^{-1}\sqrt{\frac{8}{11}}$.

(c) The equation of line BD is $\frac{x-1}{3} = \frac{y-1}{-4} = \frac{z-2}{6}$

(d) The perpendicular distance from D to the plane ABC is $\frac{80}{\sqrt{110}}$.

14. A box contain N coins, m of which are fair and rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. The probability that the coin drawn is fair, is

(a) $\frac{9m}{8N+m}$ (b) $\frac{m}{8N+m}$

(c) $\frac{N}{8n+m}$ (d) $\frac{1}{m}$

15. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α' ?

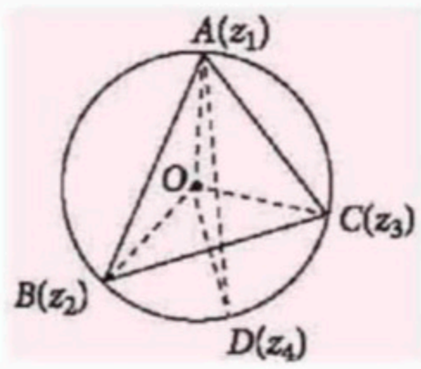
(a) abscissae of vertices

(b) abscissae of foci

(c) eccentricity

(d) directrix

16. If $A(z_1)$, $B(z_2)$, $C(z_3)$ are the vertices of a triangle ABC inscribed in the circle $|z| = 2$. Internal angle bisector of the angle A, meet the circumcircle again at $D(z_4)$, then



(a) $z_4^2 = z_2z_3$

(b) $z_4 = \frac{z_2z_3}{z_1}$

(c) $z_4 = \frac{z_1z_2}{z_3}$

(d) $z_4 = \frac{z_1z_3}{z_2}$

17. The domain of the derivative of the function

$$f(x) = \begin{cases} \tan^{-1} x, & \text{if } |x| \le 1 \\ \frac{1}{2}(|x| - 1), & \text{if } |x| > 1 \end{cases}$$ is

(a) $R - \{0\}$ (b) $R - \{1\}$

(c) $R - \{-1\}$ (d) $R - \{-1, 1\}$

18. Let $h(x) = \min\{x, x^2\}$ for every real number x. Then

(a) h is continuous for all x.

(b) h is differentiable for all x.

(c) $h'(x)=1$, for all $x > 1$.

(d) h is not differentiable at two values of x.

19. If $f(x) = \int_x^{x^2} \frac{dt}{(\log t)^2}, x \neq 0, x \neq 1$, then $f(x)$ is

(a) monotonically increasing in $(2, \infty)$

(b) monotonically increasing in $(1, 2)$

(c) monotonically decreasing in $(2, \infty)$

(d) monotonically decreasing in $(0, 1)$

20. Triangles $A_1A_2A_3$ and $B_1B_2B_3$ have side lengths (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively satisfying the relation

$$\sqrt{a_1 + a_2 + a_3}\sqrt{b_1 + b_2 + b_3} = \sqrt{a_1b_1} + \sqrt{a_2b_2} + \sqrt{a_3b_3},$$

then which one of the following statements is/are true ?

(a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$

(c) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are similar

(d) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are congruent

SECTION-II

Comprehension Type

Paragraph for Q. No. 21 to 23

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

21. The ratio of the areas of the triangles PQS and PQR is

(a) $1 : \sqrt{2}$ (b) $1 : 2$ (c) $1 : 4$ (d) $1 : 8$.

22. The radius of the circumcircle of the triangle PRS is

(a) 5 (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

23. The radius of the incircle of the triangle PQR is

(a) 4 (b) 3 (c) 8/3 (d) 2

Paragraph for Q. No. 24 and 25

Let $\vec{r}$ is a position vector of a variable point in cartesian plane OXY such that $\vec{r} \cdot (10\hat{j} - 8\hat{i} - \vec{r}) = 40$ and $p_1 = \max\{|\vec{r} + 2\hat{i} - 3\hat{j}|^2\}$, $p_2 = \min\{|\vec{r} + 2\hat{i} - 3\hat{j}|^2\}$.

24. p_2 is equal to

(a) 9 (b) $2\sqrt{2} - 1$
(c) $6\sqrt{2} + 3$ (d) $9 - 4\sqrt{2}$

25. $p_1 + p_2$ is equal to

(a) 2 (b) 10 (c) 18 (d) 5

SECTION-III

Matrix-Match Type

26. Match the following :

Column-I		Column-II	
(A)	If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p)	$\pi/6$
(B)	If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$2\pi/3$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(r)	$\pi/3$
		(s)	π

27. Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$, then match the following :

Column-I		Column-II	
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, then $f(x)$ satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	(s)	$f(x) < 1$

SECTION-IV

Numerical Value Type

28. Value of $y = 0.64^{\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots \text{upto } \infty\right)}$ is _____.

29. The exponent of 7 in ${}^{100}C_{50}$ is _____.

30. If $f(a) = 2, f'(a) = 1, g(a) = -1$ and $g'(a) = 2$, then the value of $\lim\limits_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is _____.

31. Find the value of 'a' for which $\sum\limits_{k=1}^{n} f(a+k) = 2n(32 + n)$, where the function f satisfies the relation $f(x + y) = f(x) + f(y)$ for all natural numbers x, y and further $f(1) = 4$.

32. If A is the area of the figure bounded by the straight lines $x = 0$ and $x = 2$, and the curves $y = 2^x$ and $y = 2x - x^2$, then the value of $672\left(\frac{3}{\log 2} - A\right)$ is _____.

33. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations $(y+z)\cos 3\theta = (xyz)\sin 3\theta$; $x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$; $(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$ have a solution (x_0, y_0, z_0) with $x_0 y_0 z_0 \neq 0$, is _____.

34. A line L is drawn from (4, 3) to meet the lines $L_1 : 3x + 4y + 5 = 0$ and $L_2 : 3x + 4y + 15 = 0$ at points A and B respectively. From A, a line perpendicular to L is drawn meeting the line L_2 at A_1. Similarly, from point B, a line perpendicular to L is drawn meeting the line L_1 at B_1. Thus parallelogram AA_1BB_1 is formed. Least value of area of parallelogram AA_1BB_1 is _____.

35. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is _____.

SOLUTIONS

1. (a, b, c, d) : Given, $f_n(\theta)=\frac{\sin(\theta/2)}{\cos(\theta/2)}\times\frac{1+\cos\theta}{\cos\theta}\times (1+\sec2\theta)...(1+\sec2^n\theta)$

$=\frac{\sin(\theta/2)\times2\cos^2(\theta/2)}{\cos(\theta/2)\cos\theta}(1+\sec2\theta)...(1+\sec2^n\theta)$

$=\tan\theta(1+\sec2\theta)(1+\sec4\theta)...(1+\sec2^n\theta)=\tan2^n\theta$

$\Rightarrow f_2(\pi/16)=f_3(\pi/32)=f_4(\pi/64)=f_5(\pi/128)$

$=\tan\pi/4=1$

2. (a, b) : Eccentricity of the hyperbola is $\sqrt{2}$ as it is a rectangular hyperbola. So, eccentricity e of the ellipse is $1/\sqrt{2}$.

Let the equation of the ellipse be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2=a^2(1-e^2)=a^2/2\Rightarrow a^2=2b^2$

So equation of the ellipse is $x^2+2y^2=a^2$

Let (x_1, y_1) be a point of intersection of the ellipse and the hyperbola.

$\therefore 2x_1^2-2y_1^2=1$ and $x_1^2+2y_1^2=a^2$ ...(i)

Equations of the tangents at (x_1, y_1) to the two conics are $2xx_1-2yy_1=1$ and $xx_1+2yy_1=a^2$

Since the two conics intersect orthogonally.

$\therefore \left(\frac{x_1}{y_1}\right)\left(-\frac{x_1}{2y_1}\right)=-1\Rightarrow x_1^2=2y_1^2$

and from (i), we get $x_1^2=1$, $a^2=2$.

Hence the equation of the ellipse is $x^2+2y^2=2$ and its foci are given by

$(\pm ae,0)=\left(\pm\sqrt{2}\times\frac{1}{\sqrt{2}},0\right)=(\pm1,0).$

3. (d) : Given, n is an even positive integer so n is of the form $2K$, $K\in N$.

Let $f(2K)=\frac{2K!K!}{(2K)!}\left[C_0^{\,2}-2C_1^{\,2}+3C_2^{\,2}+...\right.$

$\left....+(-1)^{2K}(2K+1)C_{2K}^2\right]$...(i)

Reversing the order of terms, we get

$f(2K)=\frac{2K!K!}{(2K)!}\left[(2K+1)C_0^{\,2}-2KC_1^{\,2}+.....+C_{2K}^2\right]$...(ii)

Adding (i) and (ii), we get

$2f(2K)=\frac{2K!K!}{(2K)!}(2K+2)(C_0^{\,2}-C_1^{\,2}+C_2^{\,2}-.....+C_{2K}^2)$

$\Rightarrow 2f(2K)=\frac{2}{{}^{2K}C_K}2(K+1)(-1)^K\,{}^{2K}C_K$

$\therefore f(n)=(-1)^{n/2}\left(\frac{n}{2}+1\right)\cdot2=(-1)^{n/2}(n+2).$

4. (b, d): $-1<1+\frac{1}{x}<1\Rightarrow -2<\frac{1}{x}<0$

$\Rightarrow \frac{1}{x}>-2$ and $\frac{1}{x}<0\Rightarrow\frac{1+2x}{x}>0$ and $x<0$

$\Rightarrow x<-\frac{1}{2}$ or $x>0$ and $x<0$ $\therefore x\in\left(-\infty,-\frac{1}{2}\right)$

Here $S=3+5\left(1+\frac{1}{x}\right)+7\left(1+\frac{1}{x}\right)^2+...$

$\left(1+\frac{1}{x}\right)S=3\left(1+\frac{1}{x}\right)+5\left(1+\frac{1}{x}\right)^2+...$

$-\frac{S}{x}=3+2\left(1+\frac{1}{x}\right)+2\left(1+\frac{1}{x}\right)^2+....$

$=3+\frac{2\left(1+\frac{1}{x}\right)}{1-\left(1+\frac{1}{x}\right)}=3+\frac{2\left(1+\frac{1}{x}\right)}{-\frac{1}{x}}$

$\Rightarrow S=2x^2-x$

5. (c) : We must have $0<2+e^{3\{x\}}\le1$

or $e^2\le2+e^{3\{x\}}\le e^3$

$\Rightarrow -2<e^{3\{x\}}\le-1$ or $e^2-2\le e^{3\{x\}}\le e^3-2$

$\Rightarrow x\in\phi$ or $\frac{1}{3}\ln(e^2-2)\le\{x\}\le\frac{1}{3}\ln(e^3-2)$

Hence, $x\in\bigcup_{n\in I}\left[n+\frac{1}{3}\ln(e^2-2),n+\frac{1}{3}\ln(e^3-2)\right]$

6. (b, c, d) : Let $P^2=[m_{ij}]_{n\times n}$

Now $m_{ij}=\sum_{l=1}^{n}p_{il}\,p_{lj}=\sum_{l=1}^{n}\omega^{i+l}\,\omega^{l+j}=\omega^{i+j}\sum_{l=1}^{n}\omega^{2l}$

$=\omega^{i+j}\cdot\omega^2[1+\omega^2+\omega^4+...+\omega^{2n-2}]$

Now, P^2 is a null matrix when n is a multiple of 3.

Hence, $n\ne3k\Rightarrow P^2\ne O$

Thus, n can be 55, 56 or 58.

7. (c) : There are two possibilities:

(i) The digits used are 1, 1, 1, 1, 1, 2, 3.

The number of numbers formed $=\frac{7!}{5!}=42$

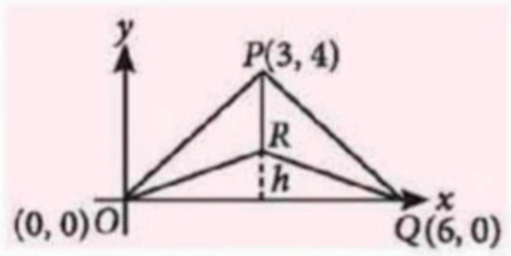
(ii) The digits used are 1, 1, 1, 1, 2, 2, 2

The number of numbers formed $=\frac{7!}{4!3!}=\frac{7\cdot6\cdot5}{6}=35$

The total number of numbers = 42 + 35 = 77.

8. (c) : ΔOPQ is an isosceles triangle, where $OP = PQ$.

So, R will lie on $x = 3$.



Now, area $\Delta ORQ = \frac{1}{3} \times$ Area ΔOPQ

$\Rightarrow \frac{1}{2} \times 6 \times h = \frac{1}{3} \times 6 \times \frac{4}{2} \Rightarrow h = \frac{4}{3}$ units.

$\therefore$ Ordinate of $R = \frac{4}{3}$.

Required point is $R\left(3, \frac{4}{3}\right)$ *i.e.*, the centroid of the triangle.

Also, Area (ΔORP) = Area $(\Delta RPQ) = \frac{1}{3} \times$ Area (ΔOPQ)

9. (b) : Let us denote the expression as A.

$$\therefore \quad A = \left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + \ldots \text{ upto 10 terms}$$

$$= \frac{8^2}{5^2} + \frac{12^2}{5^2} + \ldots. \text{ upto 10 terms}$$

$$= \frac{4^2}{5^2}(2^2 + 3^2 + 4^2 \ldots. \text{ upto 10 terms})$$

$$= \frac{16}{25}\left(\frac{11 \cdot 12 \cdot 23}{6} - 1\right) = \frac{16}{25} \times 505 = \frac{16}{5} \times 101$$

As the sum is given to be $\frac{16}{5}m \quad \therefore \quad m = 101$

10. (a) : The given equation can be written as

$\sin^{-1} 6x = -\pi/2 - \sin^{-1} 6\sqrt{3}x$

$\Rightarrow \quad \sin(\sin^{-1} 6x) = \sin(-\pi/2 - \sin^{-1} 6\sqrt{3}x)$

$\Rightarrow \quad 6x = -\cos(\sin^{-1} 6\sqrt{3}x) \quad \Rightarrow 6x = -\sqrt{1 - 108x^2}$

Squaring both sides, we get

$36x^2 = 1 - 108x^2 \Rightarrow 144x^2 = 1 \Rightarrow x = \pm 1/12$

Substituting $x = -1/12$ in the given equation, we get

$$\text{L.H.S.} = \sin^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{2} = \text{R.H.S.}$$

Therefore, $x = -1/12$ is a root of the given equation. Again, substituting $x = 1/12$ in the given equation, we get L.H.S. $= \pi/2$, so that $x = 1/12$ is an extraneous root of the given equation. Hence, $x = -1/12$ is the only root.

11. (a, b, d) : The given equation is $y_1(x^2y^3 + xy) = 1$

or $\frac{dx}{dy} = x^2y^3 + xy$

$\Rightarrow \quad x^{-2}\frac{dx}{dy} - x^{-1}y = y^3$, which is reducible to linear form.

Putting $x^{-1} = u$, we get $\frac{du}{dy} + yu = -y^3$.

The I.F. of this equation is $e^{y^2/2}$. So the solution is

$$ue^{y^2/2} = -\int y^3 e^{y^2/2}\, dy + C = -(y^2e^{y^2/2} - 2e^{y^2/2}) + C$$

$$\Rightarrow \quad 1/x = (2 - y^2) + Ce^{-y^2/2}$$

$$\Rightarrow \quad (1 - 2x)/x = -y^2 + Ce^{-y^2/2}$$

$$\Rightarrow \quad e^{y^2/2}\left(\frac{1 - 2x}{x} + y^2\right) = C$$

12. (b, c) : Let $\vec{b} = x\hat{i} + y\hat{j}$. Since $\vec{a}$ is perpendicular to $\vec{b}$ so $4x + 3y = 0$. Thus $\vec{b} = x(\hat{i} - \frac{4}{3}\hat{j})$. Let $\vec{c} = u\hat{i} + v\hat{j}$ be the required vector. According to the given condition

$$1 = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \Rightarrow 4u + 3v = 5.$$

Also, $2 = \frac{\vec{c} \cdot \vec{b}}{|\vec{b}|} \Rightarrow \frac{ux - (4/3)vx}{\sqrt{x^2(1 + (16/9))}} = 2$

$\Rightarrow 3u - 4v = \pm 10$

Solving these equations we have $u = 2$ and $v = -1$ or $u = -2/5$, $v = 11/5$.

13. (b, d) : D.r.'s of AB are 1, 4, 2 and that of CD are 2, 1, 9.

Angle between AB and CD

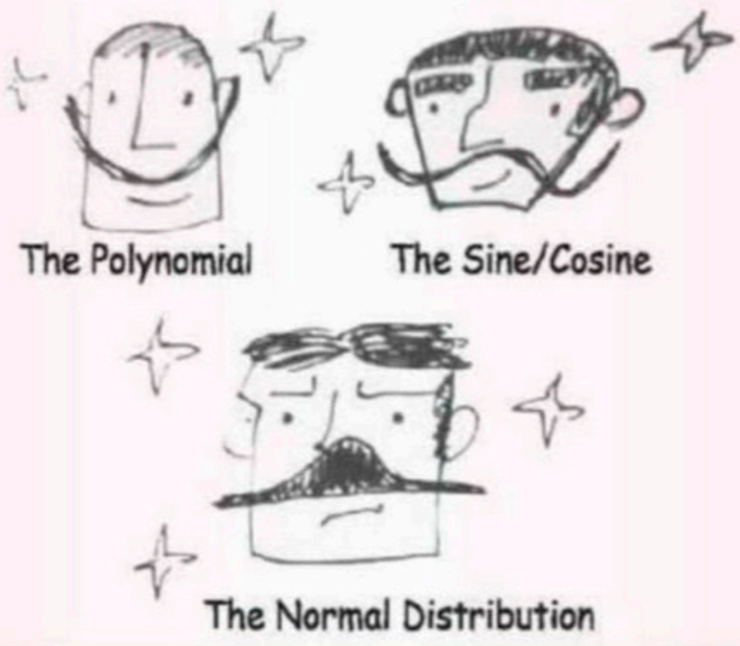
$$= \cos^{-1}\left(\frac{1 \cdot 2 + 4 \cdot 1 + 2 \cdot 9}{\sqrt{1^2 + 4^2 + 2^2}\sqrt{2^2 + 1^2 + 9^2}}\right) = \cos^{-1}\left(\frac{24}{\sqrt{1806}}\right)$$

Equation of plane ABC is $a(x - 2) + b(y - 3) + c(z - 4) = 0$.

Since it passes though B and C.

COMIC CAPSULE

Math Moustaches



$\therefore \quad a = 2k, b = -5k, c = 9k \Rightarrow 2x - 5y + 9z - 25 = 0$

Now, D.r.'s of plane ABC are 2, –5, 9 and d.r.'s of line AD are 4, 0, 8.

$\therefore$ Angle between AD and plane $ABC = \sin^{-1}\sqrt{\frac{8}{11}}$

D.r.'s of BD are 3, –4, 6. So equation of BD is

$$\frac{x-1}{3} = \frac{y+1}{-4} = \frac{z-2}{6}$$

Perpendicular distance from D to plane ABC is given by

$$\left|\frac{2(-2)-5(3)+9(-4)-25}{\sqrt{2^2+(-5)^2+9^2}}\right| = \frac{80}{\sqrt{110}}$$

14. (a) : Let E_1, E_2 and A denote the following events:

E_1 : coin selected is fair

E_2 : coin selected is biased

A : the first toss results in a head and the second toss results in a tail.

$$P(E_1) = \frac{m}{N}, P(E_2) = \frac{N-m}{N},$$

$$P(A/E_1) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}, P(A/E_2) = \frac{2}{3}\times\frac{1}{3} = \frac{2}{9}$$

$\therefore$ By Bayes' rule

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \frac{9m}{8N+m}$$

15. (b) : The given equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

$\Rightarrow \quad a = \cos\alpha, b = \sin\alpha$

$$\Rightarrow \quad e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\tan^2\alpha} = \sec\alpha \Rightarrow ae = 1$$

$\therefore$ Foci $(\pm 1, 0)$

$\therefore$ Foci remain constant with change in α.

16. (a) : We have, $\angle BOD = 2\angle BAD = \angle A$

and $\angle COD = 2\angle CAD = \angle A$

From the rotation about the point O, we get

$$\frac{z_4}{z_2} = e^{iA}, \frac{z_3}{z_4} = e^{iA} \Rightarrow z_4^2 = z_2 z_3$$

17. (d) : Given, $f(x) = \begin{cases} \tan^{-1}x, & \text{if } |x| \le 1 \\ \frac{1}{2}(|x|-1), & \text{if } |x| > 1 \end{cases}$

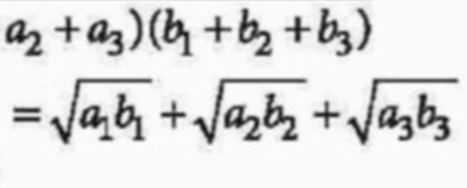
$$\text{or } f(x) = \begin{cases} \frac{1}{2}(-x-1), & -\infty < x < -1 \\ \tan^{-1}x, & -1 \le x \le 1 \\ \frac{1}{2}(x-1), & 1 < x < \infty \end{cases}$$

$\Rightarrow \quad f(x)$ is discontinuous at $x = -1$ and $x = 1$.

$\therefore$ Domain of $f'(x)$ is $R - \{-1, 1\}$.

18. (a, c, d): We have,

$$h(x) = \begin{cases} x, & \text{if } x \le 0 \\ x^2, & \text{if } 0 \le x \le 1 \\ x, & \text{if } x \ge 1 \end{cases}$$



From the graph, $h(x)$ is continuous for all $x \in R$, $h'(x) = 1$ for all $x > 1$ and h is not differentiable at $x = 0$ and 1.

19. (a, d) : We have, $f(x) = \int_x^{x^2} \frac{dt}{(\log t)^2}$

$$\therefore f'(x) = \frac{2x}{(\log x^2)^2} - \frac{1}{(\log x)^2} = \frac{1}{(\log x)^2}\left[\frac{x}{2} - 1\right]$$

$\Rightarrow \quad f'(x) > 0$ when $x \in (2, \infty)$ and $f'(x) < 0$ when $x \in (0, 2) - \{1\}$

20. (a, c) : We have, $\sqrt{(a_1+a_2+a_3)(b_1+b_2+b_3)}$

$$= \sqrt{a_1b_1} + \sqrt{a_2b_2} + \sqrt{a_3b_3}$$

Let $a_1 = A_1^2, a_2 = A_2^2, a_3 = A_3^2$

and $b_1 = B_1^2, b_2 = B_2^2, b_3 = B_3^2$

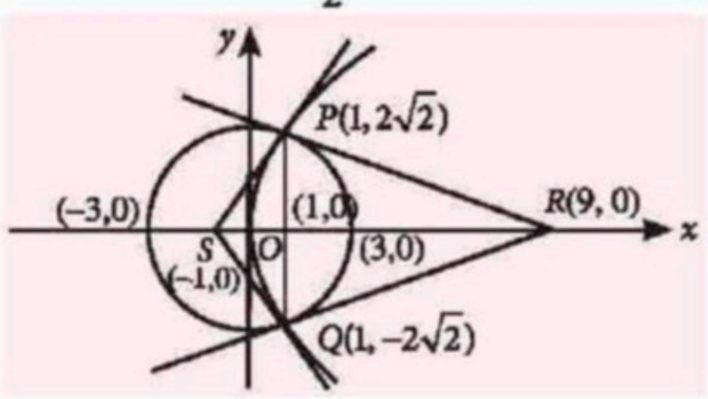
$$\therefore (A_1^2+A_2^2+A_3^2)(B_1^2+B_2^2+B_3^2) = (A_1B_1+A_2B_2+A_3B_3)^2$$

$$\Rightarrow (A_1B_2 - A_2B_1)^2 + (A_2B_3 - A_3B_2)^2 + (A_3B_1 - A_1B_3)^2 = 0$$

$$\Rightarrow \frac{A_1}{B_1} = \frac{A_2}{B_2} = \frac{A_3}{B_3} = \lambda$$

$$\text{or } \frac{\sqrt{a_1}}{\sqrt{b_1}} = \frac{\sqrt{a_2}}{\sqrt{b_2}} = \frac{\sqrt{a_3}}{\sqrt{b_3}} \quad \therefore \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

21. (c) : Area of $\Delta PQR = \frac{1}{2} \times 4\sqrt{2} \times 8 = 16\sqrt{2}$ sq.units



Area of $\Delta PQS = \frac{1}{2}\times 2 \times 4\sqrt{2} = 4\sqrt{2}$ sq. units

$$\therefore \quad \frac{\text{area}(\Delta PQS)}{\text{area}(\Delta PQR)} = \frac{1}{4}$$

22. (b) : Let $\angle PSR$ be θ.

Now, $\sin\theta = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

Also, $PR = 2R \sin\theta$

$\Rightarrow 6\sqrt{2} = 2R \times \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow R = 3\sqrt{3}$ units

23. (d): Incentre of ΔPQR is

$$I = \left(\left(\frac{6\sqrt{2}\times 1 + 4\sqrt{2}\times 9 + 6\sqrt{2}\times 1}{6\sqrt{2}+4\sqrt{2}+6\sqrt{2}} \right), \left(\frac{6\sqrt{2}\times 2\sqrt{2} + 4\sqrt{2}\times 0 + 6\sqrt{2}\times(-2\sqrt{2})}{6\sqrt{2}+4\sqrt{2}+6\sqrt{2}} \right) \right) = (3, 0)$$

Now, incircle will touch the mid-point of PQ *i.e.*, $(1, 0)$.

$\therefore$ Radius = 2 units

(24 - 25) : 24. (d) 25. (c)

Let $\vec{r} = x\hat{i} + y\hat{j}$ so $x^2 + y^2 + 8x - 10y + 40 = 0$,

centre $C(-4, 5)$, radius $= 1$, $p_1 = \max\{(x+2)^2 + (y-3)^2\}$

$p_2 = \min\{(x+2)^2 + (y-3)^2\}$

Let $P \equiv (-2, 3)$ and M be any point on circle.

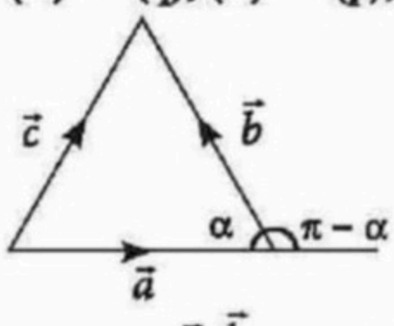
$\therefore CP = 2\sqrt{2}$, and $CM = 1$

$p_1 = (CP + CM)^2 = (2\sqrt{2}+1)^2 = 9 + 4\sqrt{2}$

$p_2 = (CP - CM)^2 = (2\sqrt{2}-1)^2 = 9 - 4\sqrt{2}$, $p_1 + p_2 = 18$

26. (A) $\rightarrow$ (q); (B) $\rightarrow$ (p); (C) $\rightarrow$ (s)

(A)



$$\cos(\pi - \alpha) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1+3}{\sqrt{1+3}\sqrt{1+3}} = \frac{2}{4} = \frac{1}{2}$$

$$\cos\alpha = -\frac{1}{2} \Rightarrow \alpha = \frac{2\pi}{3}$$

(B) Using Leibnitz rule, we get $f(b) - 3b = -2b$

$\therefore \quad f(b) = b \Rightarrow f(\pi/6) = \pi/6$

(C) Let $I = \int_{7/6}^{5/6} \sec(\pi x)dx = \left[\frac{\ln|\sec\pi x + \tan\pi x|}{\pi} \right]_{7/6}^{5/6}$

$$= \frac{\ln\left|-\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right| - \ln\left|-\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right|}{\pi} = \frac{\ln 3}{\pi}$$

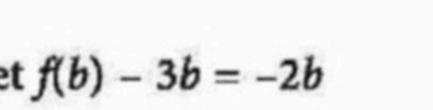
Now $\frac{\pi^2}{\ln 3} I = \frac{\pi^2}{\ln 3} \cdot \frac{\ln 3}{\pi} = \pi$

27. (A) $\rightarrow$ (p, r, s); (B) $\rightarrow$ (q, s); (C) $\rightarrow$ (q, s); (D) $\rightarrow$ (p, r, s)

$$f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6} = \frac{(x-5)(x-1)}{(x-3)(x-2)}$$

Critical points are 1, 2, 3, 5.

Sign of $f(x)$ are shown as :



(A) For $-1 < x < 1$, $f(x) > 0$.

Now, $f(x) < 1$ for $\frac{x^2 - 6x + 5}{x^2 - 5x + 6} < 1 \Rightarrow \frac{-x-1}{x^2 - 5x + 6} < 0$

$\Rightarrow \frac{x+1}{(x-2)(x-3)} > 0$

Thus, $f(x) < 1$ for $-1 < x < 2 \cup x > 3$.

Therefore, for $-1 < x < 1$, $f(x) < 1$ *i.e.*, $0 < f(x) < 1$

(B) For $1 < x < 2$, $f(x) < 0 \Rightarrow f(x) < 1$

(C) For $3 < x < 5$, $f(x) < 0 \Rightarrow f(x) < 1$

(D) For $x > 5$, $f(x) > 0$ and $f(x) < 1$.

28. (0.8): Let $S = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots = \frac{1/3}{1 - 1/3} = \frac{1}{2}$

$\therefore \quad y = (0.64)^{\log_{0.25}(1/2)}$

$\Rightarrow \log y = \frac{\log(1/2)}{\log(0.25)} \log(0.64)$

$= \frac{-\log 2}{-\log 4} \log(0.64) = \frac{1}{2}\log(0.64) \Rightarrow y = 0.8.$

PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

24×		19+			
2−		12×	2×		
			8+	48×	
4−	4÷				5×
	1−		180×		
6+		1−			

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

CONCEPT MAP

PROBABILITY

Class XI

Some Important Definitions

- **Random Experiment :** An experiment whose outcome cannot be predicted with certainty.
- **Sample Space :** The set of all outcomes of an experiment.
- **Sample Point :** Elements of sample space.

Probability

- Probability of an event(E) is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of outcomes}}$$

- Let S be the sample space containing outcomes $x_1, x_2, ..., x_n$. Then

(i) $0 \le P(x_i) \le 1$ (ii) $\sum P(x_i) = 1 \ \forall \ x_i \in S$

(iii) For any event A, $P(A) = \sum P(x_i) \ \forall \ x_i \in A$.

The number $P(x_i)$ is called probability of the outcome x_i.

Addition Theorem

- If A and B be any two events in a sample space S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- For n events $A_1, A_2, ..., A_n$:

$$P(A_1 \cup A_2 \cup ... \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) +$$

$$\sum_{i<j<k} P(A_i \cap A_j \cap A_k) - ... + (-1)^{n-1} P(A_1 \cap A_2 \cap ... \cap A_n)$$

Note :

- If A and B are mutually exclusive events, then $A \cap B = \phi$ and hence $P(A \cap B) = 0$.

$\therefore \quad P(A \cup B) = P(A) + P(B)$

- For n mutually exclusive events,

$P(A_1 \cup A_2 \cup ... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$

Booley's Inequality

- If events $E_1, E_2, ..., E_n$ are associated with a random experiment, then

(i) $P(\bigcup_{i=1}^{n} E_i) \le \sum_{i=1}^{n} P(E_i)$

(ii) $P\left(\bigcap_{i=1}^{n} E_i\right) \ge \sum_{i=1}^{n} P(E_i) - (n-1)$

Event

- A subset of the sample space associated with a random experiment is called event.
- **Types of Events**
  - **Impossible and Sure Events :** The empty set ϕ is called an impossible event and sample space S is called the sure event.
  - **Simple Event :** Event having only one sample point of a sample space.
  - **Compound Event :** Event having more than one sample point.
  - **Mutually Exclusive and Exhaustive Events :** If $E_i \cap E_j = \phi$ $\forall \ i \ne j$ and $\bigcup_{i=1}^{n} E_i = S$, then events $E_1, E_2, ..., E_n$ are called mutually exclusive and exhaustive events.

Note :

- Probability of an impossible event is 0 and probability of any sure event is 1.

Algebra of Events

Let S be sample space and A, B be any subsets of S.

- **Complementary event :** $A' = S - A$
- **Event A or B :** Set $A \cup B$
- **Event A and B :** Set $A \cap B$
- **Event A and not B :** Set $A - B = A \cap B'$

Some Important Results

- $P(\text{atleast one of } A, B \text{ occurs}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(\text{exactly one of } A, B \text{ occurs}) = P(A) + P(B) - 2P(A \cap B)$
- $P(\text{atleast one of } A, B, C \text{ occurs}) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- $P(\text{atleast two of } A, B, C \text{ occurs}) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- $P(\text{exactly two of } A, B, C \text{ occurs}) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- $P(\text{exactly one of } A, B, C \text{ occurs}) = P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

Odds in Favour and Odds Against the Event

- If a of the outcomes are favourable to an event E and b of the outcomes are against it, then odds in favour of $E = \frac{a}{b}$, odds in against of $E = b/a$

INDEFINITE INTEGRAL

CONCEPT MAP

Class XII

INDEFINITE INTEGRAL

- If $f(x)$ be a function, then the collection of all its primitives, is called the indefinite integral of $f(x)$ and is denoted by $\int f(x)dx$.

Methods

Integration by Substitution

- The process of evaluating an integral by reducing it to standard form by a proper substitution ($x = g(t)$) is called integration by substitution.

Integration by Parts

- If u and v are two differentiable functions of x, then
$\int (uv)dx = u\int vdx - \int u'\left(\int vdx\right)dx$
- General preference for first function = ILATE (Inverse, Logarithmic, Algebraic, Trigonometric, Exponential)

Some Standard Integrals

- $\int dx = x + C$
- $\int e^x dx = e^x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int a^x dx = \frac{a^x}{\log_e a} + C, a > 0$
- $\int \frac{1}{x} dx = \log_e |x| + C, x \neq 0$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \tan x dx = \log|\sec x| + C = -\log|\cos x| + C$
- $\int \cot x dx = \log|\sin x| + C$
- $\int \sec x dx = \log|\sec x + \tan x| + C$
- $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + C$

where 'C' is the constant of integration.

Properties of Integrals

- $\int f'(x)dx = f(x) + C$
- $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
- $\int kf(x)dx = k\int f(x)dx$, k being any real number.

Integrals of Some Particular Functions

- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{a^2 + x^2}\right| + C$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$
- $\int \sqrt{a^2 + x^2}dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + C$
- $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int e^x(f(x) + f'(x))dx = e^x f(x) + C$

Integration by Partial Fractions

- If $\deg f(x) \geq \deg g(x)$, then we divide $f(x)$ by $g(x)$

$\therefore \quad \frac{f(x)}{g(x)} = \text{Quotient} + \frac{p(x)}{g(x)}$, where $\deg p(x) < \deg g(x)$

Now, $\int \frac{p(x)}{g(x)}dx$ (proper fraction) is evaluated by using partial fraction decomposition.

Form of Rational Function	Form of Partial Function
$\frac{cx+d}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{cx+d}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{cx^2+dx+e}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{cx^2+dx+e}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
$\frac{cx^2+dx+e}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$, where $x^2 + bx + c$ cannot be factorised further

29. (0) : We have ${}^{100}C_{50} = \dfrac{100!}{50!50!}$.

The exponent of 7 in 50! is

$$\left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right]=7+1=8,$$

and the exponent of 7 in 100! is

$$\left[\frac{100}{7}\right]+\left[\frac{100}{7^2}\right]=14+2=16$$

Thus, exponent of 7 in ${}^{100}C_{50}$ is $\dfrac{7^{16}}{7^8 \cdot 7^8} = 7^0$

30. (5) : $\lim\limits_{x\to a} \dfrac{g(x)f(a)-g(a)f(x)}{x-a}$

$$= \lim_{x\to a} \frac{g(x)(f(a)-f(x))+f(x)(g(x)-g(a))}{x-a}$$

$$=-\lim_{x\to a} g(x) \lim_{x\to a} \frac{f(x)-f(a)}{x-a} + \lim_{x\to a} f(x) \lim_{x\to a} \frac{g(x)-g(a)}{x-a}$$

Since $g'(a)$ exists, g is continuous at $x = a$. Also, $f'(a)$ exists, so that f is continuous at $x = a$.

Hence, $\lim\limits_{x\to a} g(x)=g(a)=-1$ and $\lim\limits_{x\to a} f(x) = f(a) = 2$.

Thus $\lim\limits_{x\to a} \dfrac{g(x)f(a)-g(a)f(x)}{x-a}=-g(a)f'(a)+f(a)g'(a)$

$= -(-1)1 + 2 \times 2 = 5$

31. (15.5): $f(n)=f\underbrace{(1+\ldots+1)}_{n\text{-times}}=\underbrace{f(1)+f(1)+\ldots+f(1)}_{n\text{-times}}$

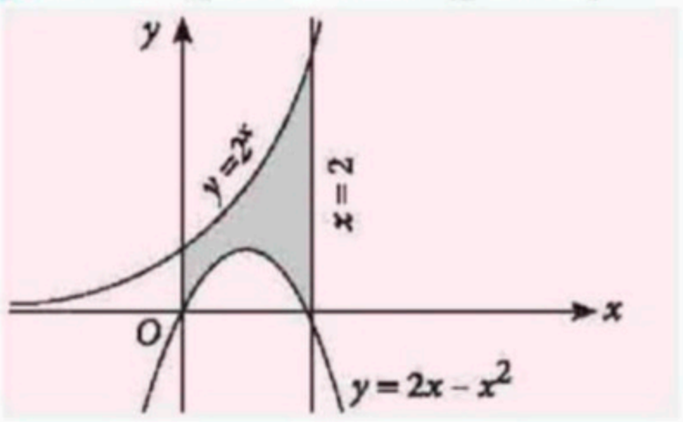
$= 4n$, for every natural number n.

The given relation can be written as

$$\sum_{k=1}^{n} 4(a+k)=2n(32+n) \Rightarrow 4an+2n(n+1)=2n(32+n)$$

$\Rightarrow \quad a = 15.5$

32. (896): The required area is given by



$$A=\int_0^2 [2^x-(2x-x^2)]\,dx=\left[\frac{2^x}{\log 2}\right]_0^2-\left[x^2-\frac{x^3}{3}\right]_0^2$$

$$= 4(\log 2)^{-1}-(\log 2)^{-1}-\left(4-\frac{8}{3}\right)=3(\log 2)^{-1}-\frac{4}{3}.$$

33. (3) : The given equations are

$(xyz)\ \sin3\theta - y\cos3\theta - z\cos3\theta = 0$

$(xyz)\ \sin3\theta - 2y\sin3\theta - 2z\cos3\theta = 0$

$(xyz)\ \sin3\theta - y(\cos3\theta + \sin3\theta) - 2z\cos3\theta = 0,$

where $xyz \neq 0$

Since the given equations has non-trivial solution which gives

$$\begin{vmatrix} \sin3\theta & -\cos3\theta & -\cos3\theta \\ \sin3\theta & -2\sin3\theta & -2\cos3\theta \\ \sin3\theta & -(\cos3\theta+\sin3\theta) & -2\cos3\theta \end{vmatrix}=0$$

$\Rightarrow \quad \sin3\theta\cos3\theta(\sin3\theta - \cos3\theta) = 0$

If $\sin3\theta = 0$, then $xyz = 0$ (not possible),

$\cos3\theta = 0$ (not possible)

If $\sin3\theta = \cos3\theta \Rightarrow \tan3\theta = 1$

$$\Rightarrow \quad 3\theta = n\pi+\frac{\pi}{4}, n\in Z$$

$$\Rightarrow \quad \theta=\frac{n\pi}{3}+\frac{\pi}{12};\ \theta=\frac{\pi}{12},\frac{5\pi}{12},\frac{9\pi}{12}$$

Thus, there are 3 solutions.

34. (8) : Here, L_1 and L_2 are parallel lines.

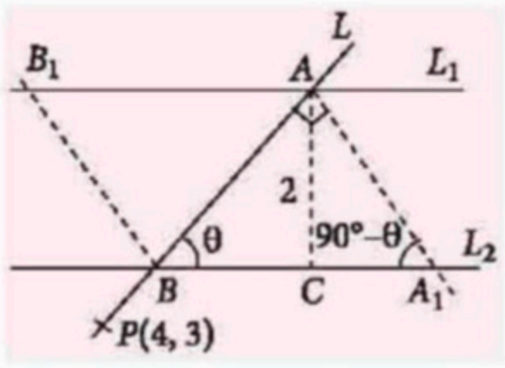
Since line L divides this parallelogram in two triangles of equal area.

Distance between two lines L_1 and L_2 is $\dfrac{|15-5|}{5} = 2$

Now, $AB = 2$ cosec θ and $AA_1 = 2$ sec θ

$\therefore$ Area of parallelogram $AA_1BB_1 = AB \times AA_1$

$$= \frac{4}{\sin\theta\cos\theta} = \frac{8}{\sin2\theta}$$



For area to be least, $\sin 2\theta = 1$ so, $\theta = 45°$.

So, least area = 8 sq. units

35. (0.4) : The number of ways that all 5 men sit adjacent to their wives = $n(A) = 4!\cdot 2^5$

The number of ways that all the American men sit adjacent to their wives = $n(S) = 5!\cdot 2^4$

$$\text{Required probability} = P(A) = \frac{n(A)}{n(S)} = \frac{4!\cdot 2^5}{5!\cdot 2^4} = \frac{2}{5} = 0.4.$$

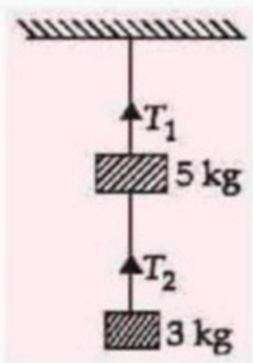
FULL LENGTH PRACTICE PAPER

BITSAT

Exam date: 16th to 25th May 2020

SECTION-I (PHYSICS)

1. Two masses of 5 kg and 3 kg are suspended with the help of a massless inextensible strings as shown in figure. The whole system is going upwards with an acceleration of 2 m s^{-2}. The tensions T_1 and T_2 are (Take $g = 10$ m s^{-2})



(a) 96 N, 36 N (b) 36 N, 96 N
(c) 96 N, 96 N (d) 36 N, 36 N

2. The dimensions of $\frac{a}{b}$ in the equation, $P = \frac{a - t^2}{bx}$, where P is pressure, x is distance and t is time are

(a) $[M^2LT^{-3}]$ (b) $[ML^0T^{-2}]$
(c) $[ML^3T^{-1}]$ (d) $[MLT^{-3}]$

3. A particle is projected from ground at some angle with the horizontal. Let P be the point at maximum height H. At what height above the point P the particle should be aimed to have range equal to maximum height?

(a) H (b) $2H$ (c) $H/2$ (d) $3H$

4. The dimensions of $\frac{\text{[Angular momentum]}}{\text{[Magnetic moment]}}$ are

(a) $[M^3LT^{-2}A^2]$ (b) $[MA^{-1}T^{-1}]$
(c) $[ML^2A^{-2}T]$ (d) $[M^2L^{-3}AT^2]$

5. A wire of length l and mass m is bent in the form of a rectangle $ABCD$ with $\frac{AB}{BC} = 2$. The moment of inertia of this wire frame about the side BC is

(a) $\frac{11}{252}ml^2$ (b) $\frac{8}{203}ml^2$
(c) $\frac{5}{136}ml^2$ (d) $\frac{7}{162}ml^2$

6. A body is rolling down an inclined plane. If kinetic energy of rotation is 40% of kinetic energy in translatory state, then the body is a

(a) ring (b) cylinder
(c) hollow ball (d) solid ball

7. For two vectors $\vec{A}$ and $\vec{B}$, $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ is always true when

(a) $|\vec{A}| = |\vec{B}| \neq 0$ (b) $\vec{A} \perp \vec{B}$
(c) $|\vec{A}| = |\vec{B}| \neq 0$ and $\vec{A}$ and $\vec{B}$ are parallel
(d) $|\vec{A}| = |\vec{B}| \neq 0$ and $\vec{A}$ and $\vec{B}$ are antiparallel

8. An ideal massless spring S can be compressed 2 m by a force of 200 N. This spring is placed at the bottom of the frictionless inclined plane which makes an angle $\theta = 30°$ with the horizontal. A 20 kg mass is released from rest at the top of the inclined plane and is brought to rest momentarily after compressing the spring 4 m. Through what distance does the mass slide before coming to rest?

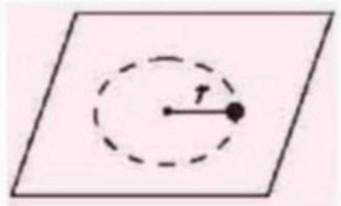
(a) 2.2 m (b) 4 m (c) 8.17 m (d) 1.9 m

9. Six moles of O_2 gas is heated from 20°C to 35°C at constant volume. If specific heat capacity at constant pressure is 8 cal mol^{-1} K^{-1} and $R = 8.31$ J mol^{-1} K^{-1}. What is the change in internal energy of the gas?

(a) 180 cal (b) 300 cal (c) 360 cal (d) 540 cal

10. A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will

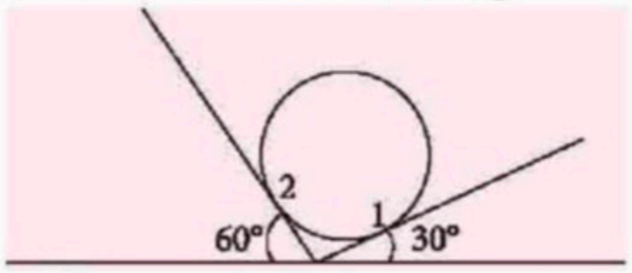
(a) decrease by a factor of 2
(b) remain constant
(c) increase by a factor of 2
(d) increase by a factor of 4



11. A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum value of u so that the particle does not return back to the earth, is

(a) $\sqrt{\frac{2GM}{R^2}}$ (b) $\sqrt{\frac{2GM}{R}}$ (c) $\sqrt{\frac{2gM}{R^2}}$ (d) $\sqrt{2gR^2}$

12. A solid sphere of mass 10 kg is placed over two smooth inclined planes as shown in figure. Normal reaction at 1 and 2 will be (Take $g = 10\ m\ s^{-2}$)



(a) $50\sqrt{3}$ N, 50 N (b) 50 N, 50 N
(c) 50 N, $50\sqrt{3}$ N (d) 60 N, 40 N

13. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The magnitude of the gravitational potential at a point situated at $a/2$ distance from the centre, will be

(a) $\frac{GM}{a}$ (b) $\frac{2GM}{a}$ (c) $\frac{3GM}{a}$ (d) $\frac{4GM}{a}$

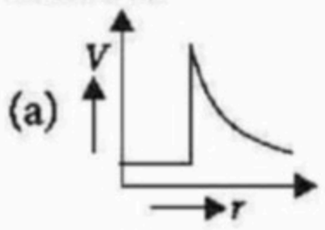
14. A mass of diatomic gas ($\gamma = 1.4$) at a pressure of 2 atmospheres is compressed adiabatically so that its temperature rises from 27°C to 927°C. The pressure of the gas in the final state is

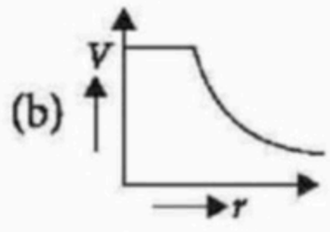
(a) 8 atm (b) 28 atm (c) 68.7 atm (d) 256 atm

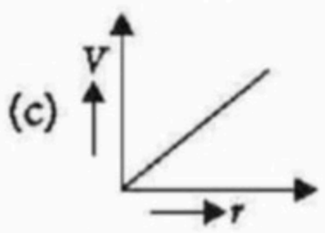
15. If 4 moles of an ideal monatomic gas at temperature 400 K is mixed with 2 moles of another ideal monatomic gas at temperature 700 K, the temperature of the mixture is

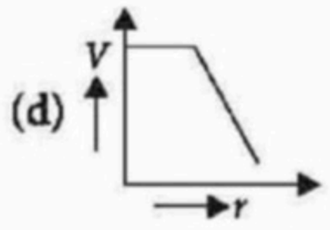
(a) 550°C (b) 500°C (c) 550 K (d) 500 K

16. In the case of a hollow metallic sphere, without any charge inside the sphere, electric potential (V) changes with respect to distance (r) from the centre as

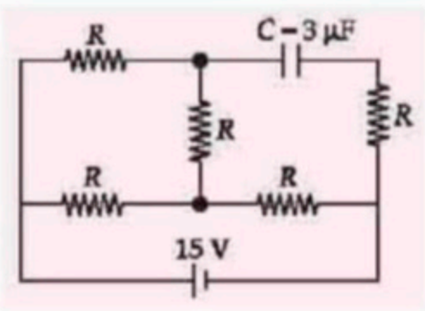
(a) 

(b) 

(c) 

(d) 

17. In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance R is of 3 Ω. The potential difference across the capacitor of capacitance 3 μF is

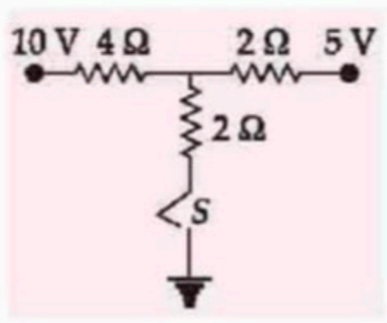


(a) Zero (b) 9 V (c) 12 V (d) 15 V

18. An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency $\frac{25}{\pi}$ Hz. At the position $x = 0.04$ m, the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is
(Potential energy is zero at mean position)

(a) 6 cm (b) 4 cm (c) 8 cm (d) 2 cm

19. As the switch S is closed in the circuit shown in figure, current passed through it is

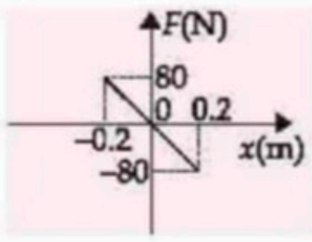


(a) Zero
(b) 1 A
(c) 2 A
(d) 1.6 A

20. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of $0.0075\ m^3$. The fundamental frequency of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become

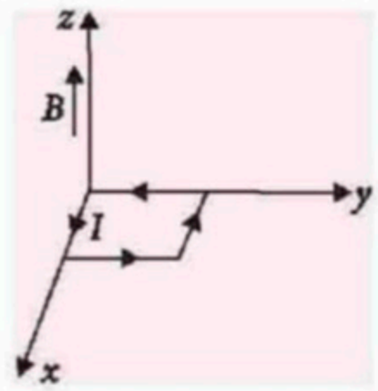
(a) 200 Hz (b) 220 Hz (c) 230 Hz (d) 240 Hz

21. A body of mass 0.01 kg executes simple harmonic motion (SHM) about $x = 0$ under the influence of a force as shown in the adjacent figure. The period of the SHM is



(a) 1.05 s (b) 0.52 s (c) 0.25 s (d) 0.03 s

22. A uniform magnetic field of 1000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop as shown in the figure?



(a) Zero (b) 1.8×10^{-2} N m
(c) 1.8×10^{-3} N m (d) 1.8×10^{-4} N m

23. A transverse wave in a medium is described by the equation $y = A \sin 2(\omega t - kx)$. The magnitude of the maximum velocity of particles in the medium is equal to that of the wave velocity, if the value of A is

(a) $\frac{\lambda}{2\pi}$ (b) $\frac{\lambda}{4\pi}$ (c) $\frac{\lambda}{\pi}$ (d) $\frac{2\lambda}{\pi}$

24. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($>> l$). The loops are coplanar and their centres coincide. What is the mutual inductance of the system?

#########

(a) $2\sqrt{2}\frac{\mu_0}{\pi}\frac{I^2}{L}$ (b) $8\sqrt{2}\frac{\mu_0}{\pi}\frac{I^2}{L}$

(c) $2\sqrt{2}\frac{\mu_0}{2\pi}\frac{I^2}{L}$ (d) None of these

25. An unpolarized light beam is incident on a surface at an angle of incidence equal to Brewster's angle. Then,

(a) the reflected and the refracted beams are both partially polarized

(b) the reflected beam is partially polarized and the refracted beam is completely polarized and are at right angles to each other

(c) the reflected beam is completely polarized and the refracted beam is partially polarized and are at right angles to each other

(d) both the reflected and the refracted beams are completely polarized and are at right angles to each other.

26. The range of voltmeter of resistance 300 Ω is 5 V. The resistance to be connected to convert it into an ammeter of range 5 A is

(a) 1 Ω in series (b) 1 Ω in parallel

(c) 0.1 Ω in series (d) 0.1 Ω in parallel

27. A square frame of side 1 m carries a current I, produces a magnetic field B at its centre. The same current is passed through a circular coil having the same perimeter as the square. The magnetic field at the centre of the circular coil is B'. The ratio B/B' is

(a) $\frac{8}{\pi^2}$ (b) $\frac{8\sqrt{2}}{\pi^2}$ (c) $\frac{16}{\pi^2}$ (d) $\frac{16}{\sqrt{2}\pi^2}$

28. A conducting circular loop is placed in a uniform magnetic field, $B = 0.025$ T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm s^{-1}. The induced emf when the radius is 2 cm, is

(a) $2\pi\,\mu$V (b) $\pi\,\mu$V (c) $\frac{\pi}{2}\,\mu$V (d) 2 μV

29. In an ac circuit, the current is $i=5\sin\left(100t-\frac{\pi}{2}\right)$ ampere and the potential difference is $V=200\sin(100t)$ volt. Then the power consumed is

(a) 200 W (b) 500 W (c) 1000 W (d) Zero

30. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath its resistance is found to be 5.5 Ω. The temperature of the hot bath is

(a) 100°C (b) 200°C (c) 300°C (d) 350°C

31. A series LCR circuit is connected to an ac source of variable frequency. When the frequency is increased continuously, starting from a small value, the power factor

(a) goes on increasing continuously

(b) goes on decreasing continuously

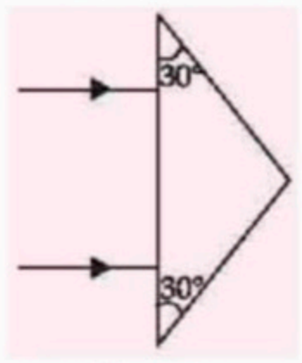
(c) becomes maximum at a particular frequency

(d) remains constant

32. A coil has 1000 turns and 500 cm^2 as its area. The plane of the coil is placed perpendicular to a uniform magnetic field of 2×10^{-5} T. The coil is rotated through 180° in 0.2 seconds. The average emf induced in the coil, in mV is

(a) 5 (b) 10 (c) 15 (d) 20

33. Two parallel light rays are incident at one surface of a prism as shown in the figure. The prism is made of glass of refractive index 1.5. The angle between the rays as they emerge is nearly



(a) 19° (b) 37° (c) 45° (d) 49°

34. What is the conductivity of a semiconductor if electron density = 5×10^{12} cm^{-3} and hole density = 8×10^{13} cm^{-3}?
($\mu_e = 2.3\ V^{-1}\ s^{-1}\ m^2$ and $\mu_h = 0.01\ V^{-1}\ s^{-1}\ m^2$)

(a) 5.634 $\Omega^{-1}\ m^{-1}$ (b) 1.968 $\Omega^{-1}\ m^{-1}$

(c) 3.421 $\Omega^{-1}\ m^{-1}$ (d) 8.964 $\Omega^{-1}\ m^{-1}$

35. If λ_1 and λ_2 are the wavelengths of the first members of the Lyman and Paschen series respectively, then $\lambda_1 : \lambda_2$ is

(a) 1 : 3 (b) 1 : 30 (c) 7 : 50 (d) 7 : 108

36. The half life of radioactive radon is 3.8 days. The time at the end of which $\left(\frac{1}{20}\right)^{th}$ of the radon sample will remain undecayed is

(a) 3.8 days (b) 16.5 days

(c) 33 days (d) 76 days

37. A hydrogen atom and a Li^{2+} ion are both in the second excited state. If l_H and l_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then

(a) $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$

(b) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$

(c) $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$

(d) $l_H < l_{Li}$ and $|E_H| < |E_{Li}|$

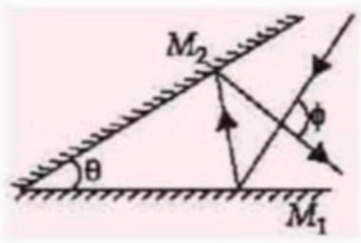
38. The ratio of de Broglie wavelength of a proton and an α particle accelerated through the same potential difference is

(a) $3\sqrt{2}$ (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) $2\sqrt{5}$

39. The temperature of the sink of a Carnot engine is 27°C and its efficiency is 25%. The temperature of the source is

(a) 227°C (b) 27°C (c) 327°C (d) 127°C

40. The reflecting surfaces of two mirrors M_1 and M_2 are at an angle θ (angle θ between 0° and 90°) as shown in the figure. A ray of light is incident on M_1. The emerging ray intersects the incident ray at an angle ϕ. Then,



(a) $\phi = \theta$ (b) $\phi = 180° - \theta$

(c) $\phi = 90° - \theta$ (d) $\phi = 180° - 2\theta$

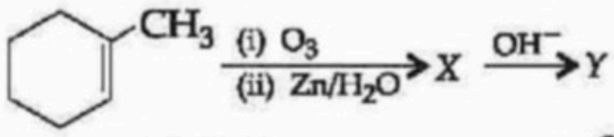
SECTION-II (CHEMISTRY)

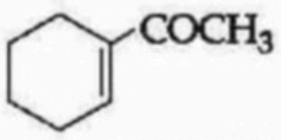
41. If λ_0 is the threshold wavelength for photoelectric emission, λ the wavelength of light falling on the surface of a metal and m the mass of the electron, then the velocity of ejected electron is given by

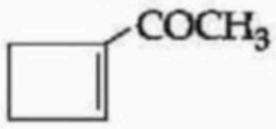
(a) $\left[\frac{2h}{m}(\lambda_0 - \lambda)\right]^{1/2}$ (b) $\left[\frac{2hc}{m}(\lambda_0 - \lambda)\right]^{1/2}$

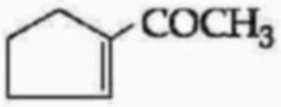
(c) $\left[\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda_0\lambda}\right)\right]^{1/2}$ (d) $\left[\frac{2h}{m}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)\right]^{1/2}$

42. Identify Y in the given reaction sequence.



(a) 

(b) 

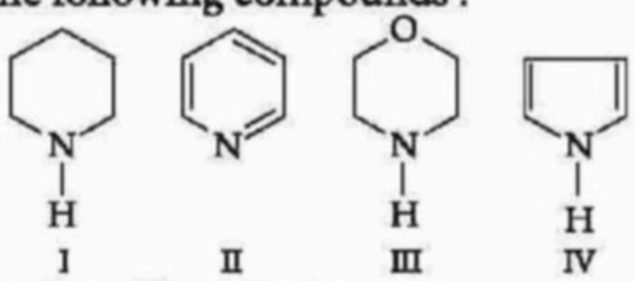
(c) 

(d) $CH_3CH_2CH_2CH_2CH_2COCH_3$

43. 0.5 g of an organic compound gave 62.2 mL of N_2 by Dumas method. Calculate the % of N in this compound.

(a) 26.33% (b) 42.72% (c) 34.52% (d) 15.55%

44. In the following compounds :



the order of basicity is

(a) IV > I > III > II (b) III > I > IV > II

(c) II > I > III > IV (d) I > III > II > IV

45. The hybridisations of atomic orbitals of nitrogen in NO_2^+, NO_3^- and NH_4^+ are

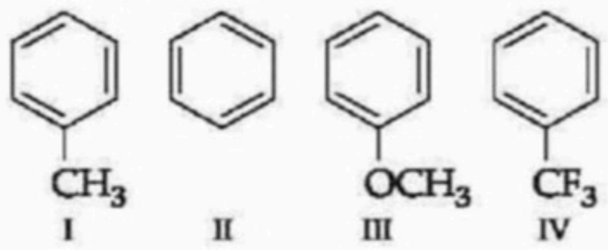
(a) sp, sp^2 and sp^3 respectively

(b) sp, sp^3 and sp^2 respectively

(c) sp^2, sp and sp^3 respectively

(d) sp^2, sp^3 and sp respectively

46. Among the following compounds, the decreasing order of reactivity towards electrophilic substitution is



(a) III > I > II > IV (b) IV > I > II > III

(c) I > II > III > IV (d) II > I > III > IV

47. At what temperature will a 5% solution (weight/volume) of glucose develop an osmotic pressure of 7 atm?

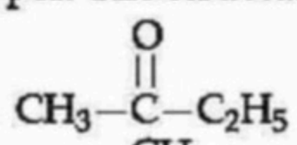
(a) 33.94 °C (b) 54.76 °C

(c) 24.55 °C (d) 47.32 °C

48. Which of the following compounds does not liberate O_2 in Bunsen burner flame?

(a) MgO (b) $NaNO_3$ (c) Pb_3O_4 (d) $KClO_3$

49. An organic compound 'X' when treated with CH_3—MgI followed by acidification gives 'Y' ($C_5H_{12}O$) which gives C_5H_{10} on heating with copper. The structure of 'X' will be

(a) 

(b) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH=O$

(c) $CH_3CH_2CH_2-CH=O$

(d) $(CH_3)_3C-CH=O$

50. Calculate q and w for the isothermal reversible expansion of one mole of an ideal gas from an initial pressure of 1.0 bar to a final pressure of 0.1 bar at a constant temperature of 273 K.

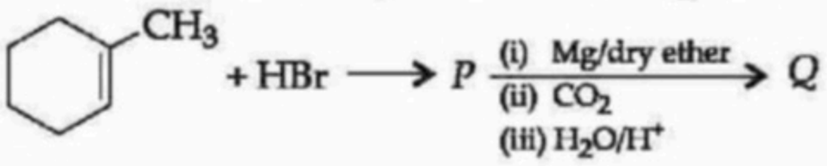
(a) 5.22 kJ, – 5.22 kJ (b) – 5.22 kJ, 5.22 kJ

(c) 27.3 kJ, – 27.3 kJ (d) – 27.3 kJ, 27.3 kJ

51. The oxidation state of Fe in Mohr's salt is

(a) + 3 (b) + 2 (c) + 6 (d) + 4

52. The final product 'Q' in the following reaction is



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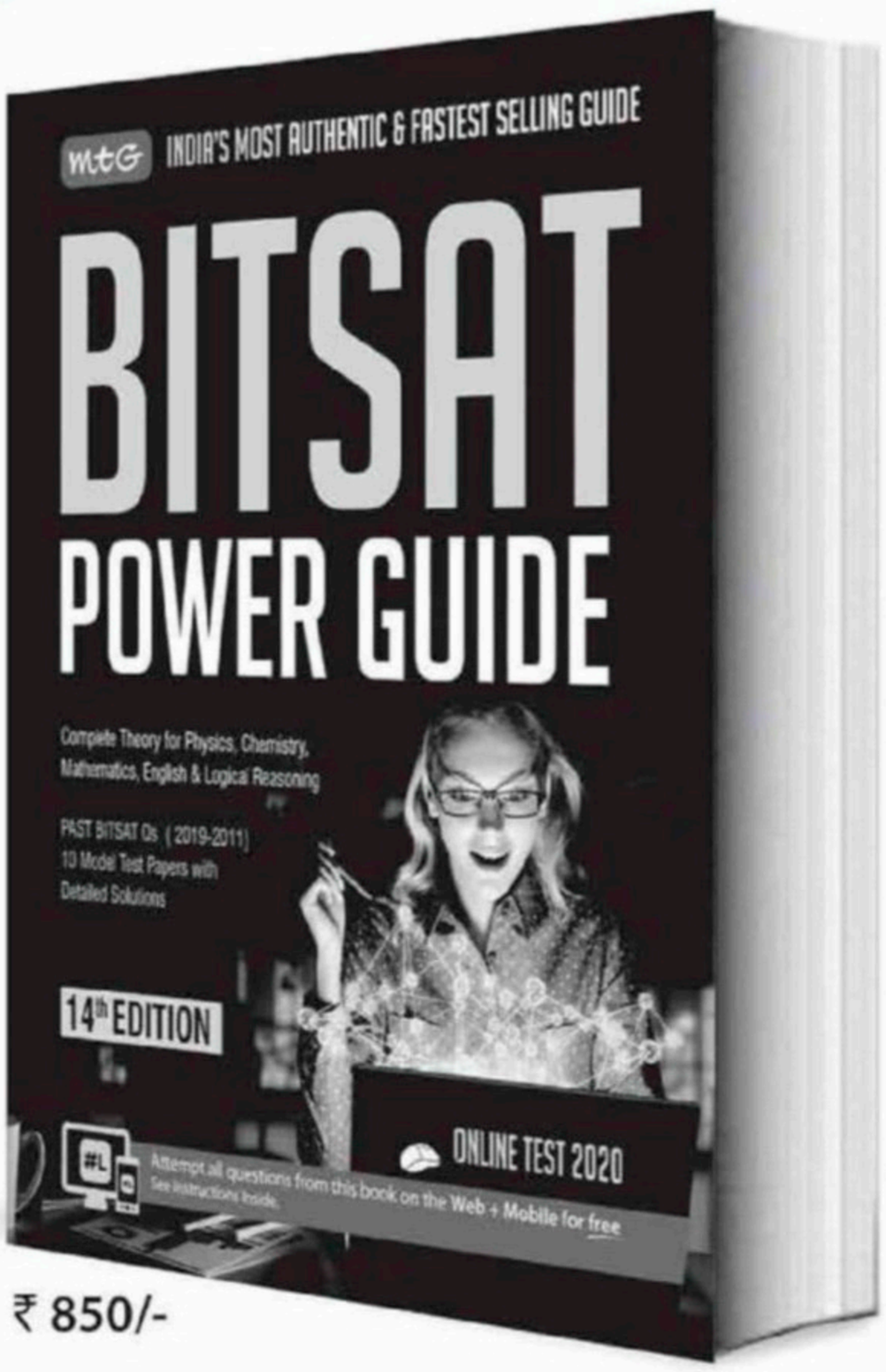

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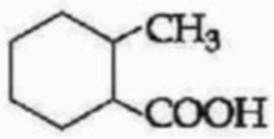
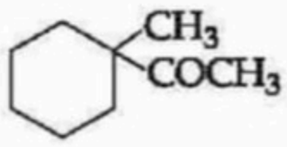
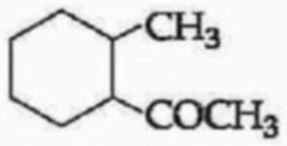
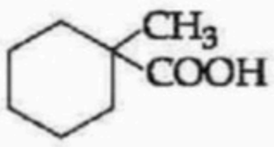
(a) 

(b) 

(c) 

(d) 

53. The increasing order of number of atoms present in the following is

(I) 52 moles of Ar (II) 52 u of He (III)52 g of He

(a) (I) < (II) < (III) (b) (III) < (II) < (I)

(c) (II) < (III) < (I) (d) (III) < (I) < (II)

54. Formation of a rose-red precipitate when a slightly alkaline solution of an inorganic salt is treated with dimethylglyoxime, confirms the presence of

(a) cobalt (b) zinc

(c) iron (d) nickel

55. To avoid the precipitation of hydroxides of Ni^{2+}, Co^{2+}, Zn^{2+} and Mn^{2+} along with those of Fe^{3+}, Al^{3+} and Cr^{3+} the third group solution should be

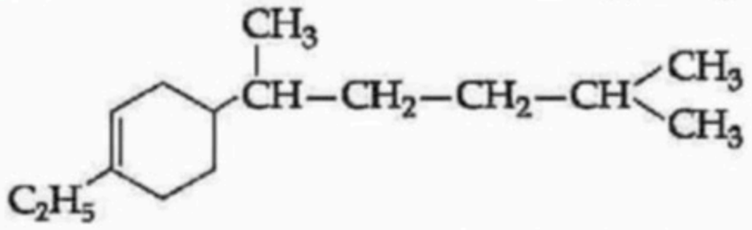
(a) heated with a few drops of conc. HNO_3

(b) treated with excess of NH_4Cl

(c) concentrated

(d) none of these

56. The IUPAC name of the following compound is



(a) 1-ethyl-4-(5-methylhexyl)cyclohex-1- ene

(b) 1-ethyl-4-(1, 4-dimethylpentyl)cyclohex-1-ene

(c) 5-(1, 4-dimethylpentyl)-2-ethylcyclohex-1-ene

(d) 4-(1, 4-dimethylpentyl)-1-ethylcyclohex-1-ene

57. Silver is monovalent and has an atomic mass of 108. Copper is divalent and has an atomic mass of 63.6. The same electric current is passed, for the same length of time through a silver coulometer and a copper coulometer. If 27.0 g of silver is deposited, then the corresponding amount of copper deposited is

(a) 63.60 g (b) 31.80 g

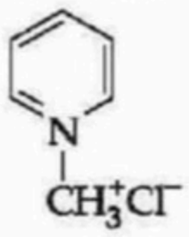
(c) 15.90 g (d) 7.95 g

58. The salts of which one of the following elements do not impart characteristic colour to the Bunsen flame?

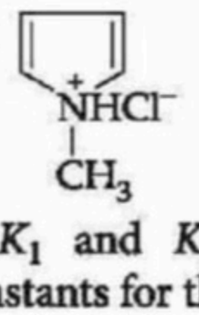
(a) Magnesium (b) Calcium

(c) Strontium (d) Sodium

59. Which one of the following is a quaternary salt?

(a) 

(b) $(CH_3)_3\overset{+}{N}HCl^-$

(c) 

(d) $(CH_3)_4\overset{+}{N}Cl^-$

60. If K_1 and K_2 are the respective equilibrium constants for the two reactions:

$XeF_{6(g)} + H_2O_{(g)} \rightleftharpoons XeOF_{4(g)} + 2HF_{(g)}$ and

$XeO_{4(g)} + XeF_{6(g)} \rightleftharpoons XeOF_{4(g)} + XeO_3F_{2(g)}$,

the equilibrium constant of the reaction:

$XeO_{4(g)} + 2HF_{(g)} \rightleftharpoons XeO_3F_{2(g)} + H_2O_{(g)}$

will be

(a) K_1/K_2^2 (b) $K_1.K_2$ (c) K_1/K_2 (d) K_2/K_1

61. The correct order of first ionisation potential among the following elements Be, B, C, N, O is

(a) B < Be < C < O < N (b) B < Be < C < N < O

(c) Be < B < C < N < O (d) Be < B < C < O < N

62. Amongst TiF_6^{2-}, CoF_6^{3-}, Cu_2Cl_2 and $NiCl_4^{2-}$ (At.nos.of Ti = 22, Co = 27, Cu = 29, Ni = 28). The colourless species are

(a) CoF_6^{3-} and $NiCl_4^{2-}$ (b) TiF_6^{2-} and CoF_6^{3-}

(c) Cu_2Cl_2 and $NiCl_4^{2-}$ (d) TiF_6^{2-} and Cu_2Cl_2

63. Identify '*Z*' in the sequence :

$$C_6H_5NH_2 \xrightarrow[273\ K]{NaNO_2 + HCl} X \xrightarrow{CuCN} Y \xrightarrow[Boil]{H^+/H_2O} Z$$

(a) C_6H_5CN (b) $C_6H_5CONH_2$

(c) C_6H_5COOH (d) $C_6H_5CH_2NH_2$

64. The oxidation number of S in Caro's acid (H_2SO_5) is

(a) + 5 (b) + 3 (c) + 2 (d) + 6

65. The solubility of sulphates in water down the IIA group follows the order Be > Mg > Ca > Sr > Ba. This is due to

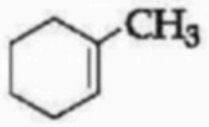
(a) increase in melting point

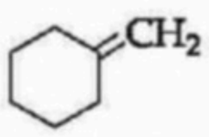
(b) increasing molecular mass

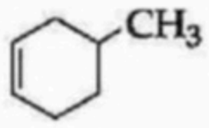
(c) decreasing lattice energy

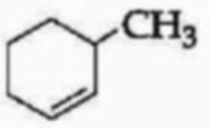
(d) high heat of solvation of smaller ions.

66. A hydrocarbon *P* of the formula C_7H_{12} on ozonolysis gives a compound *Q* which undergoes aldol condensation giving 1-acetylcyclopentene. The compound *P* is

(a) 

(b) 

(c) 

(d) 

67. Identify the pair of species in which the central atom has the same type of hybridisation.
(a) BF_3 and NCl_3 (b) H_2S and $BeCl_2$
(c) NCl_3 and H_2S (d) SF_4 and $BeCl_2$

68. The structure of *neo* pentyl group in an organic compound is

(a) $CH_3-CH_2-CH_2-CH_2-CH_2-$

(b) $CH_3-CH_2-\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{CH}}}}-CH_2-$

(c) $CH_3-\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-CH_2-$

(d) $CH_3-\underset{\displaystyle CH_3}{\overset{}{\underset{|}{CH}}}-CH_2-CH_2-$

69. The cubic unit cell of aluminium (molar mass 27.0 g mol^{-1}) has an edge length of 405 pm and density 2.70 g cm^{-3}. What type of unit cell is it?
(a) Face-centred (b) Body-centred
(c) Simple cubic (d) None of these

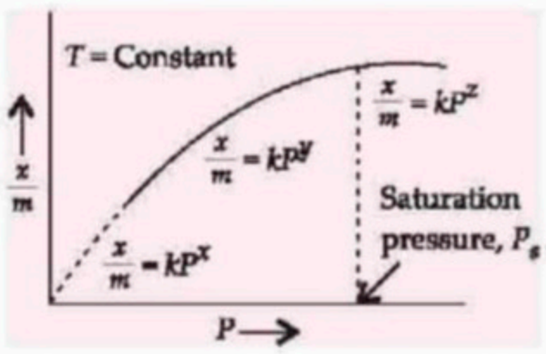
70. Al_2O_3 on heating with carbon in an atmosphere of Cl_2 at high temperature produces
(a) $Al + CO_2$ (b) $Al + CO_2 + NO$
(c) $Al_4C_3 + CO_2$ (d) $AlCl_3 + CO$

71. Which of the following does not show tautomerism?
(a) $C_6H_5COCH_3$ (b) CH_3CHO
(c) CH_3COCH_3 (d) $C_6H_5COC(CH_3)_3$

72. Which of the following has least covalent P—H bond?
(a) PH_3 (b) P_2H_6 (c) P_2H_5 (d) PH_6^+

73. What products are expected from the disproportionation reaction of hypochlorous acid?
(a) $HClO_3$ and Cl_2O (b) $HClO_2$ and $HClO_4$
(c) HCl and Cl_2O (d) HCl and $HClO_3$

74. In the given Freundlich adsorption isotherm plot, x, y and z are respectively



(a) $1, \frac{1}{n}, 0$
(b) $\frac{1}{n}, 1, 0$
(c) $0, 1, \frac{1}{n}$
(d) $0, \frac{1}{n}, 1$

75. The equilibrium constant of the reaction,

$$SO_{3(g)} \rightleftharpoons SO_{2(g)} + \frac{1}{2}O_{2(g)}$$

is 0.15 at 900 K. The equilibrium constant for

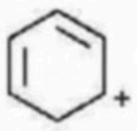
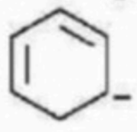
$$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$$ is
(a) 52.52 mol^{-1} L (b) 49.72 mol^{-1} L
(c) 63.34 mol^{-1} L (d) 44.44 mol^{-1} L

76. In the Hoope's process for refining of aluminium, the fused materials form three different layers and they remain separated during electrolysis also. This is because
(a) there is a special arrangement in the cell to keep the layers separate
(b) the three layers have different densities
(c) the three layers are maintained at different temperature
(d) the upper layer is kept attracted by the cathode and the lower layer is kept attracted by the anode

77. Which of the following complexes will give white precipitate with $BaCl_{2(aq)}$?
(a) $[Co(NH_3)_4SO_4]NO_2$
(b) $[Cr(NH_3)_4SO_4]Cl$
(c) $[Cr(NH_3)_5Cl]SO_4$ (d) Both (b) and (c)

78. At 25°C, the molar conductivity of 0.001 M hydrofluoric acid is 184.5 Ω^{-1} cm^2 mol^{-1}. If its Λ°_m is 502.4 Ω^{-1} cm^2 mol^{-1}, then equilibrium constant at the given concentration is
(a) 3.607×10^{-4} M (b) 5.404×10^{-4} M
(c) 2.127×10^{-4} M (d) 6.032×10^{-4} M

79. Zinc on reacting with cold, dil. HNO_3, gives
(a) $ZnNO_3$ (b) NH_4NO_3
(c) NO_2 (d) NO

80. Which of the following is aromatic?
(a)  (b)  (c) (d)

SECTION-III (ENGLISH AND LOGICAL REASONING)

81. Complete the sentence.
The telephone ..... several times before I answered it.
(a) has rung (b) was ringing
(c) would ring (d) had rung

82. Choose the correct synonym.
Audacious
(a) Manifest (b) Venture
(c) Obvious (d) Daring

83. Choose the correct antonym.
Boisterous
(a) Good (b) Happy
(c) Calm (d) Comfortable

84. Choose the one alternative which can be substituted for the given description.
One who has suddenly gained new wealth, power or prestige-
(a) Maverick (b) Parvenu
(c) Aristocrat (d) Affluent

85. Choose the correct spelling of the given words.
(a) Sacriligious (b) Sacreligious
(c) Sacrilegeous (d) Sacrilegious

86. Choose the part of the sentence that has an error.
(a) One of them (b) forget to take
(c) their mobile phone (d) from the office

PASSAGE

Direction (87-90): Read the passage and answer the questions that follow.
It is to progress in the human sciences that we must look to undo the evils which have resulted from a knowledge of the physical world hastily and superficially acquired by populations unconscious of the changes in themselves that the new knowledge has made imperative. The road to a happier world than any known in the past lies open before us if atavistic destructive passions can be kept in leash while the necessary adaptations are made. Fears are inevitable in our time, but hopes are equally rational and far more likely to bear good fruit. We must learn to think rather less of the dangers to be avoided than of the good that will lie within our grasp if we can believe in it and let it dominate our thoughts. Science, whatever unpleasant consequences it may have by the way, is in its very nature a liberator, a liberator of bondage to physical nature and is to come, a liberator from the weight of destructive passions. We are on the threshold of utter disaster or unprecedentedly glorious achievement. No previous age has been fraught with problems so momentous; and it is to science that we must look to for a happy future.

87. What does science liberate us from?
It liberates us from_____.
(a) idealistic hopes of a glorious future
(b) slavery to physical nature and from passions
(c) bondage to physical nature
(d) fears and destructive passions

88. To carve out a bright future a man should _____.
(a) cultivate a positive outlook
(b) analyse dangers that lie ahead
(c) try to avoid dangers
(d) overcome fears and dangers

89. If man's bestial yearning is controlled, _____.
(a) the future will be brighter than the present
(b) the future will be tolerant
(c) the present will be brighter than the future
(d) the present will become tolerant

90. Fears and hopes, according to the author _____.
(a) are irrational
(b) are closely linked with the life of modern man
(c) can yield good results
(d) can bear fruit

Direction (91-95): Choose the correct alternative to fill in the blanks to make a meaningful sentence.

91. Freedom and equality are the ..... rights of every human being.
(a) incalculable (b) institutional
(c) inalienable (d) inscrutable

92. This article tries to ..... us with problems of poor nations so that we help them more effectively.
(a) convince (b) project
(c) allow (d) acquaint

93. Eight scientists have ..... the national awards for outstanding contribution and dedication to the profession.
(a) bagged (b) conferred
(c) bestowed (d) picked

94. Ravi had to drop his plan of going to the picnic as he had certain ..... to meet during that period.
(a) urgencies (b) commitments
(c) preparations (d) observations

95. The speaker did not properly use the time as he went on ..... on one point alone.
(a) deliberating (b) diluting
(c) dilating (d) devoting

96. Find the missing term.
21, 25, 33, 49, 81,?
(a) 145 (b) 129 (c) 113 (d) 97

97. Find the missing number.

	5	
16	109	2
	6	

(a) 7

	21	
22	53	19
	15	

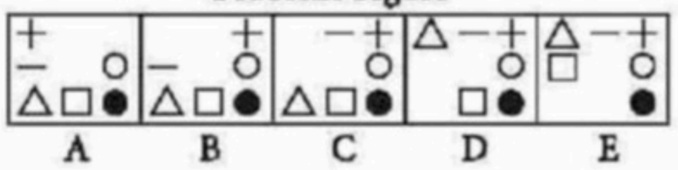
(b) 25

	51	
17	?	48
	13	

(c) 49 (d) 129

98. Select a figure from the options which will continue the same series as established by the problem figure.

Problem Figure



(a) (b)

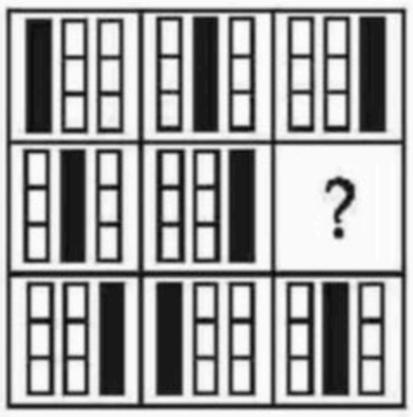
(c) (d)

99. All the components of a geometrical figure are shown in figure (X). Identify the geometrical figure from amongst the four options.

(X)

(a) (b) (c) (d)

100. Find the option which completes the given figure matrix.



(a) (b) (c) (d)

101. If 's' means '+', 't' means '×', 'u' means '÷ ' and 'v' means '–', then what is the value of 100v40s60v40u8t16?

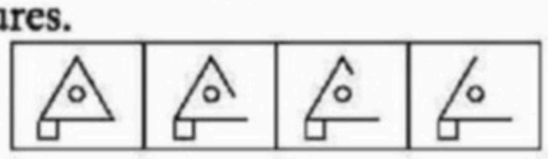
(a) 20 (b) 40 (c) 80 (d) 100

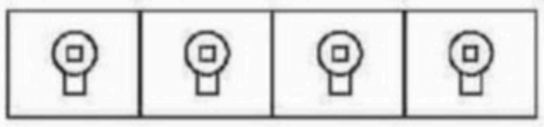
102. In a certain code, PLEADING is written as FMHCQMFB. How is SHOULDER written in that code ?

(a) KCDQTIPV (b) QDCKVPIT
(c) QDCKTIPV (d) TIPVQDCK

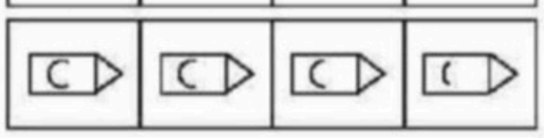
103. Select one series amongst the four series of figures, which follows the given rule.

Rule : Closed figures become more and more open figures.

(a) 

(b) 

(c)

(d) 

104. Chose out the odd numeral pair.

(a) 140 - 45 (b) 110 - 35 (c) 100 - 30 (d) 80 - 25

105. There is a certain relation between two given words on either side of :: . Identify the relation and find the missing word.

Wimbledon Trophy : Tennis : : Walker Cup : ?

(a) Hockey (b) Polo
(c) Golf (d) Wrestling

SECTION-IV (MATHEMATICS)

106. If $z_n = \cos\left(\frac{n\pi}{10}\right) + i\sin\left(\frac{n\pi}{10}\right)$, then $\prod_{n=1}^{4} z_n$ equals

(a) 0 (b) 1 (c) -1 (d) 2

107. If a, b, c, d are in G.P., then $(b-c)^2 + (c-a)^2 + (b-d)^2 =$

(a) $(c+d)^2$ (b) $(c-d)^2$ (c) $(a+d)^2$ (d) $(a-d)^2$

108. $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \ldots\ldots + C_{n-r}C_n =$

(a) $\binom{2n}{r}$ (b) $\binom{2n}{r-1}$ (c) $\binom{2n}{r+1}$ (d) $\binom{2n}{n-r}$

109. If $\Delta_r = \begin{vmatrix} 1 & n & n \\ 2r & n^2+n+1 & n^2+n \\ 2r-1 & n^2 & n^2+n+1 \end{vmatrix}$ and

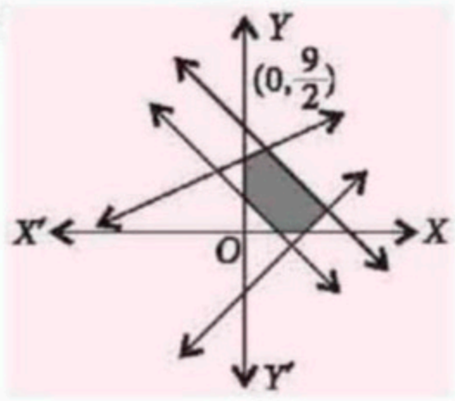
$\sum_{r=1}^{n} \Delta_r = 90$, then the value of n is

(a) 9 (b) 10 (c) 6 (d) 8

110. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6, 7 so that digits do not repeat and the terminal digits are even, is

(a) 144 (b) 72 (c) 288 (d) 720

111. Which of the following linear inequalities satisfy the shaded region of the given figure?



(a) $2x + 3y \geq 3$
(b) $3x + 4y \leq 18$
(c) $x - 6y \leq 3$
(d) All of these

112. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$

(a) 11 (b) 12 (c) 13 (d) 14

113. If $5\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 7\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\tan^{-1}\left(\frac{2x}{1-x^2}\right) - \tan^{-1}(x) = 5\pi$, then x is equal to

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

114. Distance between the lines $5x + 12y - 1 = 0$ and $10x + 24y + k = 0$ is 2, then value of k is
(a) 50 (b) –54
(c) (a) and (b) both are true
(d) none of these

115. Let $\vec{u} = \hat{i}+\hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i}+2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n} = 0$ and $\vec{v}\cdot\hat{n} = 0$, then $|\vec{w}\cdot\hat{n}|$ is equal to
(a) 1 (b) 2 (c) 3 (d) 0

116. If $y = e^x \cdot e^{x^2} \cdot e^{x^3} \ldots e^{x^n} \ldots$, for $0 < x < 1$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ equals
(a) e (b) $4e$ (c) $2e$ (d) $3e$

117. If $\int e^{2x}\cos 3x\,dx = (a\cos 3x + b\sin 3x)e^{2x} + c$, then $a + b =$
(a) $\frac{1}{5}$ (b) $\frac{6}{13}$ (c) $\frac{4}{13}$ (d) $\frac{5}{13}$

118. The equation of hyperbola referred to its axes as axes of co-ordinate whose distance between the foci is 20 and eccentricity equals $\sqrt{2}$, is
(a) $x^2 - y^2 = 25$ (b) $x^2 - y^2 = 50$
(c) $x^2 - y^2 = 125$ (d) $x^2 + y^2 = 25$

119. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to
(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) –1

120. The function $f(x) = x^2 \log x$ in the interval $[1, e]$ has
(a) a point of maxima and minima
(b) a point of maxima only
(c) no point of maxima and minima in $[1, e]$
(d) no point of maxima and minima

121. If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of the conic $9x^2 - 4y^2 = 36$, then which of the following is true ?
(a) $e_1^2 + e_2^2 = 2$ (b) $3 < e_1^2 + e_2^2 < 4$
(c) $e_1^2 + e_2^2 > 4$ (d) none of these

122. The equation of the plane through the points (1, 2, 3), (–1, 4, 2) and (3, 1, 1) is
(a) $5x + y + 12z - 23 = 0$ (b) $5x + 6y + 2z - 23 = 0$
(c) $x + 6y + 2z - 13 = 0$ (d) $x + y + z - 13 = 0$

123. The probability that Ram will be alive 30 years hence is $\frac{7}{11}$ and Shyam will be alive is $\frac{7}{10}$. The probability that both Ram and Shyam will be dead 30 years hence is
(a) $\frac{1}{9}$ (b) $\frac{12}{110}$ (c) $\frac{2}{121}$ (d) $\frac{11}{123}$

124. If $a = \hat{i}+\hat{j}+\hat{k}$, $b = 4\hat{i}+3\hat{j}+4\hat{k}$ and $c = \hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|c| = \sqrt{3}$, then
(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 1, \beta = \pm 1$
(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

125. The parametric representation of a point on the ellipse whose foci are (3, 0) and (–1, 0) and eccentricity 2/3 is
(a) $(1+3\cos\theta, \sqrt{3}\sin\theta)$
(b) $(1+3\cos\theta, 5\sin\theta)$
(c) $(1+3\cos\theta, 1+\sqrt{5}\sin\theta)$
(d) $(1+3\cos\theta, \sqrt{5}\sin\theta)$

126. Corner points of the feasible region are determined by the system of linear constraints (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q, so that the minimum of Z occurs at (3, 0) and (1, 1) is
(a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

127. If 1, $\log_{81}(3^x + 48)$ and $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P., then x is equal to
(a) 1 (b) 2 (c) 9 (d) 3

128. The shortest distance between the skew lines $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}+3\hat{j}+2\hat{k})$ and $\vec{r} = (4\hat{i}+5\hat{j}+6\hat{k}) + \mu(2\hat{i}+3\hat{j}+\hat{k})$ is
(a) 3 (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\sqrt{6}$

129. If p : 4 is an even prime number,
q : 6 is a divisor of 12 and
r : the HCF of 4 and 6 is 2, then which one of the following is true ?
(a) $(p \wedge q)$ (b) $(p \vee q) \wedge \sim r$
(c) $\sim (q \wedge r) \vee p$ (d) $\sim p \vee (q \wedge r)$

130. $\int_0^{\pi/4} \ln(1 + \tan x)dx =$
(a) $\frac{\pi}{4}\ln 2$ (b) $\frac{\pi}{8}\ln 2$ (c) $\frac{\pi}{2}\ln 2$ (d) $\ln 2$

131. The mean square deviation of n observations x_1, x_2, ..., x_n about –2 and 2 are 18 and 10 respectively. Then standard deviation of the given set is
(a) 1 (b) 2 (c) 3 (d) 4

132. If a, b, c are the roots of $y^3 - 3y^2 + 3y + 26 = 0$ and ω is cube root of unity, then the value of $\frac{a-1}{b-1} + \frac{b-1}{c-1} + \frac{c-1}{a-1}$ equals
(a) $-3\omega^2$ (b) $3\omega^2$ (c) ω^2 (d) $2\omega^2$

133. If $A+B+C=\pi$, then $\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)+\tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right)+\tan\left(\frac{C}{2}\right)\tan\left(\frac{A}{2}\right)$ is equal to

(a) $\pi/6$ (b) 3 (c) 2 (d) 1

134. If the area of the circle $4x^2+4y^2-8x+16y+k=0$ is 9π sq. units, then $k=$

(a) 4 (b) 16 (c) ± 16 (d) -16

135. The orthogonal trajectories of the family of semi-cubical parabola $ay^2=x^3$ is given by

(a) $x^2+3y^2=c^2$ (b) $3x^2+y^2=c^2$

(c) $x+3y^2=c^2$ (d) $3y^2+2x^2=c^2$

136. The equation of the plane through the intersection of the planes $x+2y+3z-4=0$ and $4x+3y+2z+1=0$ and passing through the origin is

(a) $17x+14y+11z=0$ (b) $7x+4y+z=0$

(c) $x+14y+11z=0$ (d) $x+y+11z=0$

137. The equation of family of curves for which the length of the normal is equal to the radius vector is

(a) $2y^2-x^2=k^2$ (b) $y+x=k$

(c) $y^2+x^2=k^2$ (d) $y-x=k$

138. If $\int\frac{xe^x}{\sqrt{1+e^x}}dx=(Ax+B)\sqrt{1+e^x}+C\ln\left|\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right|+D$, then $A+B+C=$

(a) 2 (b) -2 (c) 4 (d) 0

139. For the primitive integral equation $ydx+y^2dy=xdy;x\in R,y>0,y(1)=1$, then $y(-3)$ is

(a) 3 (b) 5 (c) 1 (d) 2

140. The area of the region $\{(x,y):x^2+y^2\le 1\le x+y\}$ is (in sq. units)

(a) $\frac{1}{2}$ (b) $\frac{3\pi-2}{4}$ (c) $\frac{\pi-2}{4}$ (d) $\frac{\pi}{3}$

141. In how many ways can the letters of the word DAUGHTER can be arranged so that the vowels may never be separated ?

(a) 4320 (b) 4520 (c) 4145 (d) 6545

142. Let the statement $m^2>100$, the statement $P(k+1)$ will be true if

(a) $P(1)$ is true (b) $P(2)$ is true

(c) $P(k)$ is true (d) none of these

143. If $a\in[-10,0]$, then the probability that the graph of the function $y=x^2+2(a+3)x-2a-6$ is strictly above x-axis is

(a) 3/5 (b) 2/5 (c) 1/5 (d) 4/5

144. The value of x for which the matrix $A=\begin{bmatrix}x+a & b & c\\ a & x+b & c\\ a & b & x+c\end{bmatrix}$ is non-singular, is

(a) $R-\{0,-(a+b+c)\}$ (b) $R-\{0\}$

(c) $R-\{-(a+b+c)\}$ (d) none of these

145. Domain of $f(x)=\log_2\log_2\log_2....\log_2 x$ (n times) is

(a) $(2,\infty)$ (b) $(2^n,\infty)$

(c) $(2^{n+1},\infty)$ (d) none of these

146. $\lim\limits_{x\to 1^-}\frac{\int_1^x|t-1|dt}{\sin^2(x-1)}=$

(a) 0 (b) 1 (c) 1/2 (d) $-1/2$

147. If $\vec{p}\times\vec{q}=\vec{r}$ and $\vec{q}\times\vec{r}=\vec{p}$, then

(a) $r=1,p=q$ (b) $p=1,q=1$

(c) $r=2p,q=2$ (d) $q=1,p=r$

148. The locus of the foot of the perpendicular drawn from the centre to any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) a circle (b) an ellipse

(c) a hyperbola (d) none of these

149. Coefficient of variation of two distributions are 60% and 75%, and their standard deviations are 18 and 15 respectively. Find their arithmetic means respectively.

(a) 30, 30 (b) 30, 20 (c) 20, 30 (d) 20, 20

150. A coin is tossed 10 times. The probability of getting exactly six heads is

(a) 512/513 (b) 105/512

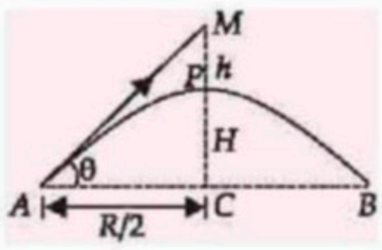
(c) 100/153 (d) ${}^{10}C_6$

SOLUTIONS

1. (a) **2.** (b)

3. (a) : From figure, $AC=R/2$, $PC=H$, $h=MP=?$

Given, $R=H$



or $\frac{u^2\sin 2\theta}{g}=\frac{u^2\sin^2\theta}{2g}$

or $\tan\theta=4$

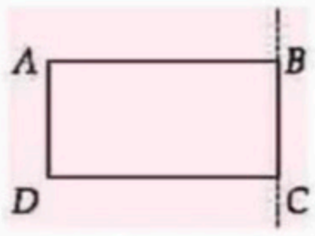
In ΔAMC, $\tan\theta=\frac{MC}{AC}=\frac{MP+PC}{AC}$

$\therefore\quad 4=\frac{h+H}{R/2}\Rightarrow 4=\frac{2(h+H)}{H}\Rightarrow h=H$

4. (b) : $\frac{[\text{Angular momentum}]}{[\text{Magnetic moment}]}=\left[\frac{2m}{q}\right]$

$$=\left[\frac{2m}{It}\right]=\left[\frac{\mathrm{M}}{\mathrm{AT}}\right]=[\mathrm{MA^{-1}\,T^{-1}}]$$

5. (d) : $\frac{AB}{BC}=2 \quad \therefore \quad AB=DC=\frac{l}{3}$



and $BC=AD=\frac{l}{6}$

Similarly, $m_{AB}=m_{DC}=\frac{m}{3}$

and $m_{BC}=m_{AD}=\frac{m}{6}$

Moment of inertia about the desired axis is

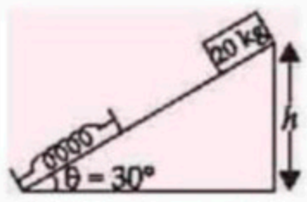
$I=I_{AB}+I_{AD}+I_{DC}+I_{BC}$

$$=\frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2+\left(\frac{m}{6}\right)\left(\frac{l}{3}\right)^2+\frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2+0=\frac{7}{162}ml^2$$

6. (d) **7. (b)**

8. (c) : As the spring is compressed by 2 m with the application of a force of 200 N, hence its spring constant k is given by

$$k=\frac{F}{2}=\frac{200\ \mathrm{N}}{2\ \mathrm{m}}=100\ \mathrm{N\,m^{-1}}$$



Suppose l be the distance along the inclined plane. Applying the conservation of energy,

$$\frac{1}{2}kx_1^2=mgh=mgl\sin\theta$$

$$\text{or}\quad \frac{1}{2}\times100\times4^2=20\times9.8\times l\times\frac{1}{2}\ \Rightarrow\ l=\frac{800}{98}=8.17\ \mathrm{m}$$

9. (d) **10. (d)**

11. (b) : According to law of conservation of mechanical energy

$$\frac{1}{2}mu^2-\frac{GMm}{R}=0\ \Rightarrow\ u^2=\frac{2GM}{R}$$

$$u=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}\qquad\left(\because\ g=\frac{GM}{R^2}\right)$$

12. (a) **13. (c)** **14. (d)**

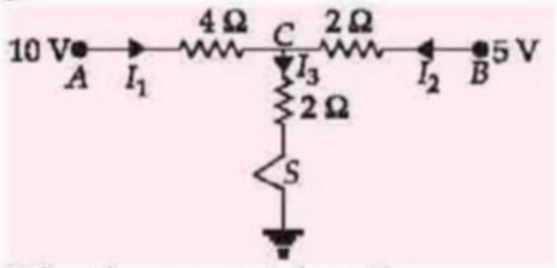
15. (d) : Here, $n_1=4,\ T_1=400\ \mathrm{K},\ n_2=2,\ T_2=700\ \mathrm{K}$

The temperature of the mixture is

$$T_{\text{mixture}}=\frac{n_1T_1+n_2T_2}{n_1+n_2}=\frac{1600\ \mathrm{K}+1400\ \mathrm{K}}{6}=500\ \mathrm{K}$$

16. (b) **17. (c)** **18. (a)**

19. (c) : The currents through various arms will be as shown in figure.



Let V be the potential at C.

Applying Kirchhoff 's first law at C, we get $I_1+I_2=I_3$

$$\frac{10-V}{4}+\frac{5-V}{2}=\frac{V-0}{2}\ \Rightarrow V=4\ \mathrm{V}\ \therefore\quad I_3=\frac{4\ \mathrm{V}}{2\ \Omega}=2\ \mathrm{A}$$

20. (d)

21. (d) : From the given graph, k = Slope of F-x graph

$$=\frac{(80-0)\ \mathrm{N}}{(0.2-0)\ \mathrm{m}}=400\ \mathrm{N\,m^{-1}}$$

Time period of SHM is $T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{0.01\ \mathrm{kg}}{400\ \mathrm{N\,m^{-1}}}}=0.03\ \mathrm{s}$

22. (a)

23. (b) : The given equation of the transverse wave is $y=A\sin2(\omega t-kx)$

Velocity of the particle $=\frac{dy}{dt}=2A\omega\cos2(\omega t-kx)$

Maximum velocity $=2A\omega$

Velocity of the wave $=\frac{\text{Coefficient of } t}{\text{Coefficient of } x}=\frac{2\omega}{2k}=\frac{\omega}{k}$

As per question,

$$2A\omega=\frac{\omega}{k}\ \text{ or }\ 2A=\frac{1}{k}=\frac{\lambda}{2\pi}\ \Rightarrow A=\frac{\lambda}{4\pi}$$

24. (a) **25. (c)** **26. (b)** **27. (d)**

28. (b) : Magnetic flux linked with the loop is

$\phi=BA\cos\theta=B(\pi r^2)\cos0^\circ=B\pi r^2$

The magnitude of the induced emf is

$$|\varepsilon|=\frac{d\phi}{dt}=\frac{d}{dt}\left(B\pi r^2\right)=B\pi2r\frac{dr}{dt}$$

$=0.025\times\pi\times2\times2\times10^{-2}\times1\times10^{-3}=\pi\times10^{-6}\ \mathrm{V}=\pi\ \mu\mathrm{V}$

29. (d) : Here, $i=5\sin\left(100t-\frac{\pi}{2}\right)\ \mathrm{A}\ ;\ V=200\sin(100t)\ \mathrm{V}$

$\therefore$ Phase difference between V and i is, $\phi=\frac{\pi}{2}$

Power consumed, $P=V_{\text{rms}}\ i_{\text{rms}}\cos\phi=V_{\text{rms}}\ i_{\text{rms}}\cos90^\circ=$ zero

30. (b) **31. (c)** **32. (b)** **33. (b)**

34. (b) : $\sigma=e[n_e\mu_e+n_h\mu_h]$

$=1.6\times10^{-19}[5\times10^{18}\times2.3+8\times10^{19}\times0.01]=1.968\ \Omega^{-1}\,\mathrm{m^{-1}}$

35. (d)

36. (b) : As $N=N_0e^{-\lambda t}$ or $\frac{N}{N_0}=e^{-\lambda t}$

Taking natural logarithm on both sides, we get

$$-\lambda t=\ln\frac{N}{N_0}\ \text{or}\ \lambda t=\ln\frac{N_0}{N}\ \text{or}\ t=\frac{1}{\lambda}\ln\frac{N_0}{N}$$

Here, $\lambda=\frac{\ln2}{T_{1/2}}=\frac{0.693}{3.8}=0.182,\ \frac{N}{N_0}=\frac{1}{20}$

$$\therefore\quad t=\frac{1}{0.182}\ln(20)=16.5\ \text{days}$$

37. (a) : In second excited state, $n=3$, So, $l_{\mathrm{H}}=l_{\mathrm{Li}}=3\left(\frac{h}{2\pi}\right)$ while $E\propto Z^2$ and $Z_{\mathrm{H}}=1,\ Z_{\mathrm{Li}}=3$

So, $|E_{\mathrm{Li}}|=9|E_{\mathrm{H}}|$ or $|E_{\mathrm{H}}|<|E_{\mathrm{Li}}|$

38. (b) : de Broglie wavelength associated with charged particle is given by,

$$\lambda = \frac{h}{\sqrt{2mqV}}, \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}}$$

As $\frac{m_p}{m_\alpha} = \frac{1}{4}$ and $\frac{q_\alpha}{q_p} = \frac{2}{1} \Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{4 \times 2} = 2\sqrt{2}$

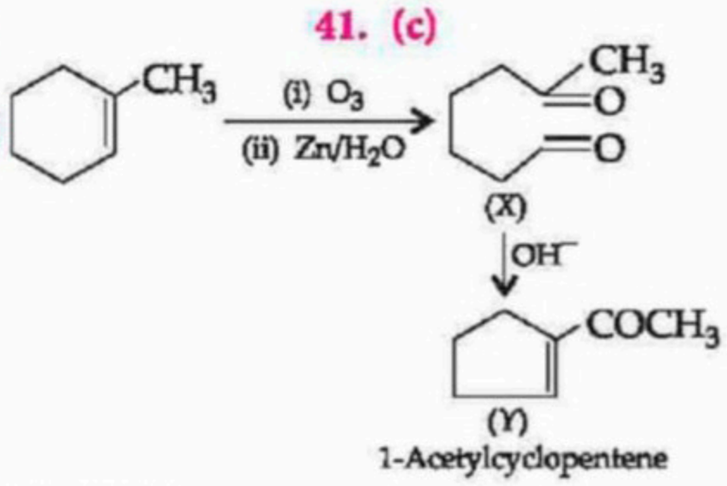
39. (d) : Efficiency of a Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

Here, $\eta = 25\%$, $T_2 = 27°C = 300$ K

$\therefore \quad \frac{25}{100} = 1 - \frac{300}{T_1} \Rightarrow T_1 = 400$ K $= 127°C$

40. (d) **41. (c)**

42. (c) :



43. (d) : % of N (By Dumas method)

$$= \frac{28 \times \text{Volume of } N_2 \text{ at STP} \times 100}{22400 \times \text{Weight of compound}} = \frac{28 \times 62.2 \times 100}{22400 \times 0.5} = 15.55\%$$

44. (d)

45. (a) : The shapes of NO_2^+, NO_3^- and NH_4^+ are linear, trigonal planar and tetrahedral, respectively. Thus, the hybridisations of atomic orbitals of nitrogen in these species are *sp*, sp^2 and sp^3 respectively.

46. (a)

47. (a) : Given; $w = 5$ g, $V = 100$ mL, $\pi = 7$ atm, $M = 180$ g mol^{-1}

Applying $\pi V = \frac{w}{M} \cdot RT \Rightarrow 7 \times \frac{100}{1000} = \frac{5}{180} \times 0.0821 \times T$

$\therefore \quad T = \frac{7 \times 100 \times 180}{1000 \times 5 \times 0.0821} = 306.94$ K or 33.94 °C

48. (a) : Magnesium oxide has high m.pt., therefore, it does not decompose in Bunsen burner flame to give O_2 gas. All other compounds decompose to give O_2 gas.

$$2Pb_3O_4 \xrightarrow{\Delta} 6PbO + O_2$$
$$2NaNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$$
$$2KClO_3 \xrightarrow{\Delta} 2KCl + 3O_2$$

49. (a) : From the preparation and molecular formula of *Y*, it is clear that *Y* is an alcohol. As *Y* on heating with copper gives hydrocarbon C_5H_{10}, hence *Y* must be 3° alcohol. This 3° alcohol having 5 carbon atoms can be obtained only when 4 carbon atoms ketone (*X*) is treated with CH_3—MgI.

50. (a)

51. (b) : Mohr's salt is $FeSO_4.(NH_4)_2SO_4.6H_2O$. Let the oxidation state of Fe be *x*.

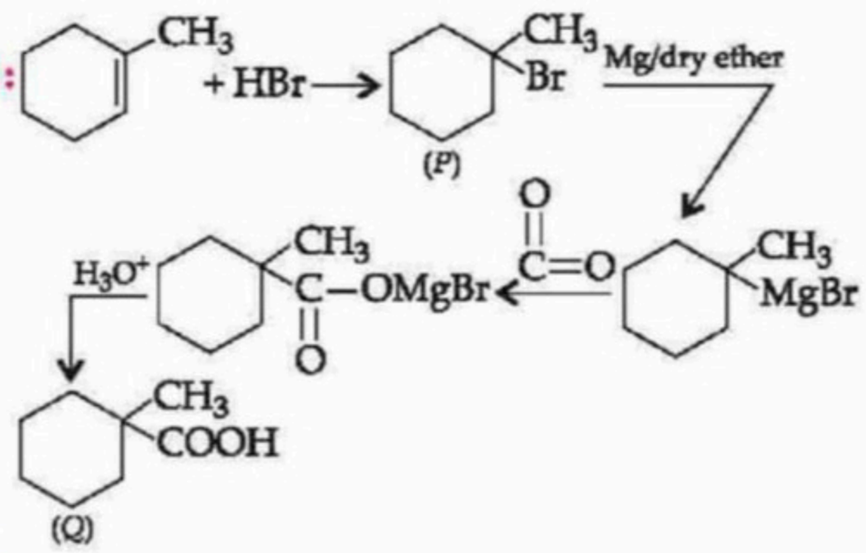
Sum of the oxidation number for $(NH_4)_2SO_4 = 0$

Sum of the oxidation number for $H_2O = 0$

Sum of the oxidation number for $SO_4^{2-} = -2$

$x + (-2) + 0 + 6 \times 0 = 0$; $x = +2$

52. (d) :

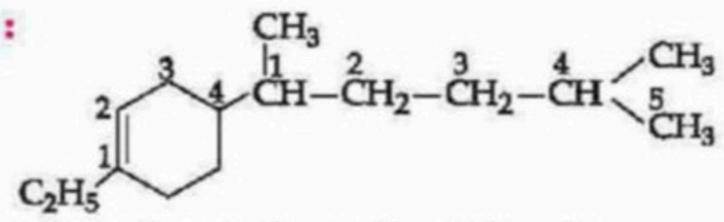


53. (c)

54. (d) : Nickel gives rose-red precipitate with dimethyl glyoxime.

55. (b)

56. (b) :



1-Ethyl-4-(1, 4-dimethylpentyl)cyclohex-1-ene

57. (d) : When the same quantity of electricity is passed through the solutions of different electrolytes, the weights of the elements deposited are directly proportional to their chemical equivalents.

i.e. $\frac{\text{Wt. of Cu deposited}}{\text{Wt. of Ag deposited}} = \frac{\text{Eq. wt. of Cu}}{\text{Eq. wt. of Ag}}$

$\frac{W_{Cu}}{27} = \frac{63.6/2}{108/1} \quad \therefore \quad W_{Cu} = \frac{63.6 \times 27}{2 \times 108} = 7.95$ g

58. (a)

59. (d) : N is attached to four carbon atoms.

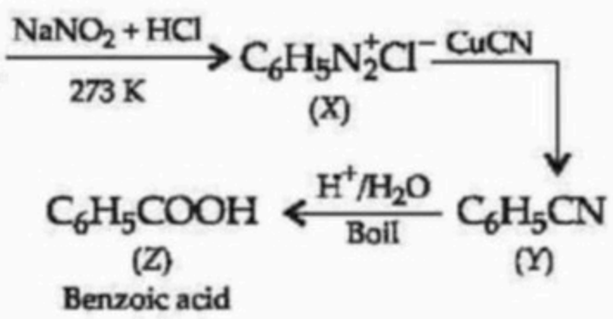
60. (d) : $K_1 = \frac{[XeOF_4][HF]^2}{[XeF_6][H_2O]}, K_2 = \frac{[XeOF_4][XeO_3F_2]}{[XeO_4][XeF_6]}$

$\frac{[XeO_3F_2][H_2O]}{[XeO_4][HF]^2} = \frac{K_2}{K_1}$

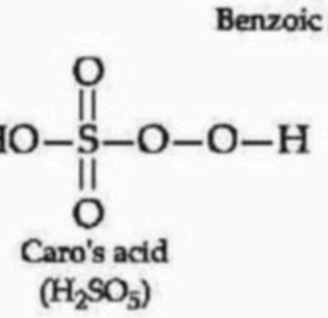
61. (a)

62. (d) : Colour of salts is a property of partially filled *d*-orbitals. Since TiF_6^{2-} has completely empty and Cu_2Cl_2 has completely filled *d*-subshells, hence these are colourless salts.

63. (c) :



64. (d) :



Two oxygen atoms are in peroxo linkage *i.e.*, oxidation number = –1

$3(-2)+2(-1)+x+2(+1)=0 \Rightarrow -8+x+2=0 \Rightarrow x=+6$

65. (d) **66. (a)**

67. (c) : N atom in NCl_3 and S atom in H_2S are sp^3 hybridised.

68. (c)

69. (a) : $\rho=\dfrac{Z\times M}{N_0\times a^3}$ or $Z=\dfrac{\rho\times N_0\times a^3}{M}$

$$Z=\frac{2.7\times6.023\times10^{23}\times(405\times10^{-10})^3}{27.0}=41$$

i.e., number of atoms per unit cell is 4. Hence, unit cell is face-centred type.

70. (d) : $Al_2O_3 + 3C + 3Cl_2 \xrightarrow{1000°C} 2AlCl_3 + 3CO$

71. (d) : $C_6H_5COC(CH_3)_3$ does not contain an α-hydrogen and hence does not show tautomerism.

72. (d) : Due to the +ve charge on P, it attracts the electrons of the P—H bond towards itself. As a result, it has some ionic character. In other words, the P—H bond in PH_6^+ is least covalent.

73. (d) : $3HClO_{(aq)} \longrightarrow HClO_{3(aq)} + 2HCl_{(aq)}$

It is a disproportionation reaction of hypochlorous acid where the oxidation number of Cl changes from +1 (in ClO^-) to +5 (in ClO_3^-) and –1 (in Cl^-).

74. (a) **75. (d)** **76. (b)**

77. (c) : Sulphate ion is present outside the coordination sphere so it can form white ppt. of $BaSO_4$ with $BaCl_{2(aq)}$.

78. (c) : $\alpha=\dfrac{\Lambda_m}{\Lambda_m^\circ}=\dfrac{184.5}{502.4}=0.367$

$$K=\frac{C\alpha^2}{1-\alpha}=\frac{0.001M\times(0.367)^2}{0.633}=2.127\times10^{-4}\ M$$

79. (b) : $4Zn + 10HNO_3 \longrightarrow 4Zn(NO_3)_2 + NH_4NO_3 + 3H_2O$ (dilute)

80. (b) : Aromaticity can be predicted by the use of Huckle's rule which says that $(4n + 2)$ π-electrons are required in delocalisation system to give it aromaticity.

81. (d) **82. (d)** **83. (c)** **84. (b)**
85. (d) **86. (c)** **87. (b)** **88. (a)**
89. (a) **90. (b)** **91. (c)** **92. (d)**
93. (a) **94. (b)** **95. (c)**

96. (a) : The pattern is +4, +8, +16, +32, ... *i.e.*, $+2^2, +2^3, +2^4, +2^5$, ...

$\therefore$ Missing number = $81 + 2^6 = 81 + 64 = 145$.

97. (b) : In 1st fig., $(16-6)^2+(5-2)^2=10^2+3^2=109$

In 2nd fig., $(22-15)^2+(21-19)^2=7^2+2^2=53$

$\therefore$ In 3rd fig., missing number = $(17-13)^2+(51-48)^2$
$= 4^2+3^2=25$

98. (d) : One of the symbols shifts by two steps in every subsequent figure, in a sequence.

99. (c)

100. (c) : Black lines moves forward one step.

101. (b) : $100 - 40 + 60 - 40 + 8 \times 16 = 40$

102. (c) : The last four letters of the word are written in the reverse order, followed by the first four letters in the same order. In the group of letters so obtained, each of the first four letters is moved one step backward while each of the last four letters is moved one step forward to get the code. Thus, we have :

SHOULDER → REDL/SHOU → QDCK/TIPV

103. (c)

104. (c) : In all other pairs, $\dfrac{1^{st}\text{ number}-5}{3}=2^{nd}$ number

105. (c) : Wimbledon Trophy is associated with the game of tennis. Similarly, Walker Cup is associated with the game of golf.

106. (c)

107. (d) : Since $b^2 = ca$, $c^2 = bd$, $bc = ad$

Now, $(b-c)^2+(c-a)^2+(b-d)^2$
$= b^2-2bc+c^2+c^2-2ca+a^2+b^2-2bd+d^2$
$= a^2+d^2-2bc = a^2+d^2-2ad = (a-d)^2$.

108. (d) : $C_0 + C_1x + C_2x^2 + ... = (1+x)^n$

$\Rightarrow C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ... + C_rx^{n-r} + ... = (x+1)^n$

Multiplying above two equations and taking the coefficients of x^{n-r}, we get

$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n$ = coefficient of x^{n-r} in $(1+x)^n(x+1)^n = (1+x)^{2n}$, which is $\binom{2n}{n-r}$.

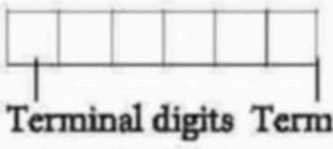
109. (a) : $\sum_{r=1}^{n}\Delta_r=\begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, we get

$\sum_{r=1}^{n}\Delta_r=\begin{vmatrix} 0 & n & n \\ -1 & n^2+n+1 & n^2+n \\ 0 & n^2 & n^2+n+1 \end{vmatrix} = n^2+n = n(n+1)$

Given that $\sum_{r=1}^{n}\Delta_r = 90 \Rightarrow n(n+1) = 90 \Rightarrow n = 9$

110. (d) :



1st place can be filled in 3 ways and 6th place in 2 ways and rest of the places in $5 \times 4 \times 3 \times 2$ ways.

$\therefore$ Required number = $3 \times 2 \times 5 \times 4 \times 3 \times 2 = 720$

111. (d)

112. (c) : $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$= 3(1-\sin 2x)^2 + 6(1+\sin 2x) + 4[(\sin^2 x+\cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)]$

$= 3(1 - 2\sin 2x + \sin^2 2x) + 6 + 6\sin 2x + 4\left[1 - \frac{3}{4}\sin^2 2x\right] = 13$

113. (d)

114. (c) : According to given condition, $\left|\frac{\frac{k}{2}+1}{13}\right| = 2$

$\Rightarrow \frac{k}{2} + 1 = \pm 26 \Rightarrow k = 50 \text{ or } -54$

115. (c) : Let $\hat{n} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, where $a_1^2 + a_2^2 + a_3^2 = 1$

Since, $\vec{u}\cdot\hat{n} = 0 \Rightarrow a_1 + a_2 = 0$

Also, $\vec{v}\cdot\hat{n} = 0 \Rightarrow a_1 - a_2 = 0$

$\therefore a_1 = a_2 = 0 \Rightarrow a_3 = 1 \text{ or } -1$

$\Rightarrow \hat{n} = \hat{k} \text{ or } -\hat{k}; \quad \therefore |\vec{w}\cdot\hat{n}| = 3$

116. (b)

117. (d) : Let $I = \int \text{Re}\, e^{(2+3i)x}dx = \text{Re}\frac{e^{(2+3i)x}}{2+3i} + c$

$= \frac{e^{2x}}{13}\text{Re}(2-3i)(\cos 3x + i\sin 3x) + c$

$= \frac{e^{2x}}{13}(2\cos 3x + 3\sin 3x) + c \quad \therefore a + b = \frac{2}{13} + \frac{3}{13} = \frac{5}{13}$

118. (b) : Given, $2ae = 20$ and $e = \sqrt{2} \quad \therefore \quad a = \frac{20}{2e} = 5\sqrt{2}$

Again, $b^2 = a^2(e^2 - 1)$; $b^2 = 25 \times 2(2 - 1) = 50$

$\therefore$ Equation of hyperbola is $x^2 - y^2 = 50$

119. (a)

120. (c) : $f(x) = x^2 \log x \Rightarrow f'(x) = x^2/x + 2x \log x$

For maxima & minima, $f'(x) = 0$

$\Rightarrow 0 = x + 2x \log x$

$\Rightarrow 2 \log x = -1$, which is not possible in $[1, e]$

$\therefore$ No point of maxima & minima exist in $[1, e]$.

121. (b) : From given equations, we have $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\therefore \quad e_1^2 = \frac{9-4}{9}$ Also, $\frac{x^2}{4} - \frac{y^2}{9} = 1 \quad \therefore \quad e_2^2 = \frac{4+9}{4}$

Now, $e_1^2 + e_2^2 = \left(\frac{5}{9}\right) + \left(\frac{13}{4}\right) = \frac{137}{36} > 3 \text{ but } < 4$

$\therefore \quad 3 < e_1^2 + e_2^2 < 4$

122. (b)

123. (b) : Given, $P(R) = P(\text{Ram will be alive}) = \frac{7}{11}$

$\therefore \quad P(R^c) = \frac{4}{11}$

Also, $P(S) = P(\text{Shyam will be alive}) = \frac{7}{10} \quad \therefore \quad P(S^c) = \frac{3}{10}$

$\therefore \quad P(R^c \cap S^c) = P(R^c)\,P(S^c) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110}$

124. (d) : Since a, b, c are linearly dependent, $[a\ b\ c] = 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \text{ or } \begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{vmatrix} = 0$

$\Rightarrow -(\beta - 1) = 0 \Rightarrow \beta = 1$

Also, $|c| = \sqrt{3} \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

125. (d)

126. (b) : We must have value of Z at $(3, 0)$ = value of Z at $(1, 1)$

$\Rightarrow 3p + 0\cdot q = 1\cdot p + 1\cdot q \Rightarrow 3p = p + q \Rightarrow p = \frac{1}{2}q$

127. (b) : The three numbers are $\log_9 9$, $\log_{9^2}(3^x + 48)$ and $\log_9\left(3^x - \frac{8}{3}\right)$,

i.e., $\log_9 9$, $\frac{1}{2}\log_9(3^x + 48)$, $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P.

$\Rightarrow \left\{(3^x + 48)^{\frac{1}{2}}\right\}^2 = 9\left(3^x - \frac{8}{3}\right) \Rightarrow 8\cdot 3^x = 72 \Rightarrow 3^x = 9 \Rightarrow x = 2$

128. (c)

129. (d) : Clearly p is false and q and r are true.

In the given options only (d) can be true because it is $\sim p \vee (q \wedge r)$ which means not p or (q and r) which means p is not correct and q and r are correct.

130. (b)

131. (c) : $\frac{1}{n}\sum_{i=1}^{n}(x_i + 2)^2 = 18$ and $\frac{1}{n}\sum_{i=1}^{n}(x_i - 2)^2 = 10$

$\therefore$ Adding and subtracting, we get

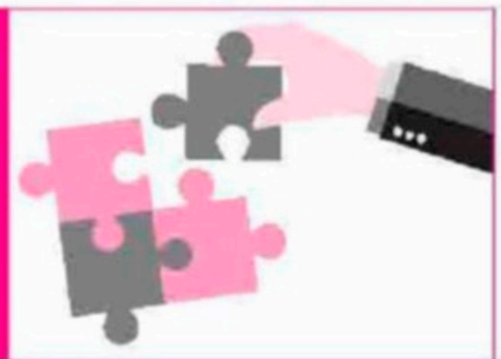
$\frac{2}{n}\sum_{i=1}^{n}(x_i^2 + 2^2) = 28$ and $\frac{8}{n}\sum_{i=1}^{n}x_i = 8$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}x_i^2 = 10$ and $\frac{1}{n}\sum_{i=1}^{n}x_i = 1$

$\therefore \quad \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2 = 10 - 1 = 9 \quad \Rightarrow \sigma = 3$

PUZZLE CORNER

ANSWER · MARCH 2020



80× 5	4	3 3	5− 1	6	2÷ 2
4	11+ 6	5	1− 2	3	1
9× 3	2 2	3− 1	4	30× 5	6
1	3	11+ 6	5	2÷ 2	4
6 6	8× 1	2	13+ 3	4	8+ 5
10× 2	5	4	6	1 1	3

132. (b) : Given equation can be written as $(y-1)^3 = -27$

$\Rightarrow \left(\frac{y-1}{-3}\right)^3 = 1 \Rightarrow \frac{y-1}{-3} = (1)^{1/3} \therefore \frac{y-1}{-3} = 1, \omega, \omega^2$

As a, b, c are the roots of given equation.

$\therefore \quad a-1 = -3, b-1 = -3\omega, c-1 = -3\omega^2$

$\therefore \frac{a-1}{b-1}+\frac{b-1}{c-1}+\frac{c-1}{a-1} = \frac{1}{\omega}+\frac{1}{\omega}+\frac{\omega^2}{\omega^3} = \omega^2+\omega^2+\omega^2 = 3\omega^2$

133. (d)

134. (d) : Equation of the circle is

$4x^2+4y^2-8x+16y+k=0$

$\Rightarrow \quad x^2+y^2-2x+4y+\frac{k}{4}=0$

$\therefore \quad \text{Radius} = \sqrt{(-1)^2+(2)^2-\frac{k}{4}} = \sqrt{5-\frac{k}{4}}$

$\text{Area} = \pi\left(\sqrt{5-\frac{k}{4}}\right)^2 = 9\pi$ (Given)

$\Rightarrow \quad \pi\left(5-\frac{k}{4}\right) = 9\pi \Rightarrow -4 = \frac{k}{4} \Rightarrow k = -16$

135. (d)

136. (a) : Required equation of plane is

$(x+2y+3z-4)+k(4x+3y+2z+1)=0 \quad \ldots$ (i)

It passes through (0, 0, 0) so $-4+k=0$ or $k=4$

Substituting $k=4$ in (i), we get

$(x+2y+3z-4)+4(4x+3y+2z+1)=0$

$\Rightarrow \quad 17x+14y+11z=0$

137. (c) : Length of normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

It is given that $y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^2+y^2}$

$\Rightarrow \quad y^2+y^2\left(\frac{dy}{dx}\right)^2 = x^2+y^2 \Rightarrow y^2\left(\frac{dy}{dx}\right)^2 = x^2$

$\Rightarrow x^2(dx)^2 = y^2(dy)^2 \Rightarrow ydy \pm xdx = 0 \Rightarrow y^2 \pm x^2 = k^2$

138. (d)

139. (a) : $\frac{ydx-xdy}{y^2} = -dy \Rightarrow d\left(\frac{x}{y}\right) = -dy$

Integrating, we get $\frac{x}{y} = -y+c$

$\because \quad y(1)=1 \Rightarrow c=2 ; y^2-2y+x=0$

For $x=-3$, we have $y^2-2y-3=0 \Rightarrow (y-3)(y+1)=0$

$\Rightarrow y = 3, -1$

140. (c)

141. (a) : In the word DAUGHTER, there are 3 vowels (A, U, E) and 5 consonants (D, G, H, T, R)

3 vowels should never be separated means that they are always together. So, the group of 3 vowels can be treated as one letter, X (say).

Hence we can arrange D, G, H, T, R, X, *i.e.*, 6 letters, taking all of them at a time in ${}^6P_6$ ways, *i.e.*, $6! = 720$ ways.

For each of these 720 ways, the 3 vowels can be arranged among themselves in 3 ! ways, *i.e.*, 6 ways.

$\therefore \quad$ Required number of ways $= 6 \times 720 = 4320$

142. (c) : $P(r)$ is true.

$\Rightarrow \quad r^2 > 100 \Rightarrow r^2+2r+1 > 100+2r+1$

$\Rightarrow \quad (r+1)^2 > 100 \Rightarrow P(r+1)$ is true as $r^2+(2r+1) > r^2 > 100$

$\Rightarrow \quad P(k+1)$ is true (say $r=k$)

$P(k+1)$ is true when every $P(k)$ is so. In order to prove that $P(k+1)$ is true. It is sufficient to consider $P(k)$ is true.

143. (c) : $f(x) = y = x^2+2(a+3)x-2a-6 \; \forall \; x \in R$

(i) $f(x) > 0$, coefficient of x^2 is $1 > 0$ (ii) $D < 0$

$\therefore \quad 4(a+3)^2-4(-2a-6) < 0$

$\Rightarrow \quad (a+3)^2+(2a+6) < 0 \Rightarrow a^2+8a+15 < 0$

$\Rightarrow \quad (a+3)(a+5) < 0 \; -5 < a < -3$

Now, $n(S)$ = Length of interval as $a \in [-10, 0]$

$n(S) = 0-(-10) = 10 \quad \therefore \quad n(S) = 10$

$n(A)$ = Length of interval, when $-5 < a < -3$

$\Rightarrow \quad n(A) = -3-(-5) = 2$

$\therefore \quad$ Required probability $= \frac{n(A)}{n(S)} = \frac{2}{10} = \frac{1}{5}$

144. (a)

145. (d)

146. (d) : $\lim\limits_{x\to 1^-} \frac{\int_1^x |t-1|dt}{\sin^2(x-1)} = \lim\limits_{x\to 1^-} \frac{\int_1^x -(t-1)dt}{\sin^2(x-1)}$

$= -\lim\limits_{x\to 1^-} \frac{[(t-1)^2]_1^x}{2\sin^2(x-1)} = -\lim\limits_{x\to 1^-} \frac{(x-1)^2-0}{2\sin^2(x-1)}$

$= -\lim\limits_{x\to 1^-} \left(\frac{1}{2}\right)\left(\frac{(x-1)^2}{\sin^2(x-1)}\right) = -\frac{1}{2}$

147. (d)

148. (d) : Given, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Equation of any tangent to the ellipse is

$y = mx+\sqrt{a^2m^2+b^2} \quad \ldots$(i)

Equation of the line through the centre (0, 0) and perpendicular to (i) is

$x+my=0 \quad \ldots$(ii)

Eliminating m from (i) and (ii), we get the required locus of the foot of the perpendicular as, $y = -\frac{x^2}{y}+\sqrt{a^2\frac{x^2}{y^2}+b^2}$

$\Rightarrow \quad (x^2+y^2)^2 = a^2x^2+b^2y^2$, which is fourth degree equation in x and y does not represent a circle, an ellipse or a hyperbola.

149. (b)

150. (b) : $n = 10$, P (success) $= 1/2$

So, P (getting exactly 6 heads) $= {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = {}^{10}C_4\frac{1}{2^{10}}$

$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1\times 2^{10}} = \frac{210}{1024} = \frac{105}{512}$

Challenging PROBLEMS ON JEE Advanced

Exam on 17th May, 2020

Single Option Correct Type

1. If $S_n = {}^nC_1 - \left(1+\frac{1}{2}\right){}^nC_2 + \left(1+\frac{1}{2}+\frac{1}{3}\right){}^nC_3$ $+.....+(-1)^{n-1}\left(1+\frac{1}{2}+....+\frac{1}{n}\right){}^nC_n$, then S_n is equal to

(a) $\frac{2^n-1}{n(n+1)}$ (b) $\frac{1}{n}$

(c) $\frac{2}{n(n+1)}$ (d) $\frac{2^n}{n(n+1)}$

2. Let A denotes the subset of {1, 5, 9, 13, ..., 1093} having the property that no two elements of A add up to 1094. If the maximum number of elements in A is a three digit number, then sum of its digits is

(a) 9 (b) 11 (c) 13 (d) 15

3. If the points (2, 3) and (3, 2) on a parabola are equidistant from the focus, the slope of its tangent at vertex is

(a) parallel to $y = x$ (b) parallel to $y = -x$

(c) parallel to x-axis (d) parallel to y-axis

4. Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros. Let s denotes the sum of the four least possible values of n. What is the sum of the digits of s?

(a) 7 (b) 8 (c) 9 (d) 10

5. For every real number x, let $f(x) = [x](21^{\{x\}} - 1)$. The set of all numbers x such that $1 \le x < 21$ and $f(x) \le 1$ is a union of disjoint intervals. The sum of the lengths of those intervals is ([·] denotes GIF, {·} denotes fractional part function)

(a) 1 (b) log21/log20

(c) log20/log19 (d) 21/20

6. The sum of all the positive solutions of

$2\cos 2x\left(\cos 2x - \cos\left(\frac{20\pi^2}{x}\right)\right) = \cos 4x - 1$ is

(a) 24π (b) 36π (c) 18π (d) 72π

7. For some particular value of N, when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables a, b, c and d, each to some positive power. The value of N is

(a) 7 (b) 14 (c) 21 (d) 28

8. The coefficients in the series $T = 2 + 8x + 18x^2 + 37x^3 +..... + a_rx^r +....., $ satisfy a recurrence relation of the form $a_{r+2} + ka_{r+1} + ya_r = 0$. An expression for infinite sum of T (assuming it exists) is given by

(a) $\frac{4+6x}{(2-x)(1+2x)}$ (b) $\frac{4+6x}{(2+x)(1+2x)}$

(c) $\frac{4+6x}{(2-x)(1-2x)}$ (d) $\frac{4-6x}{(2-x)(1+2x)}$

9. Let a, b, c, d be positive integers such that $ab = cd$, then which of these is not a possible value for $a^2 + b^2 + c^2 + d^2$?

(a) 194 (b) 195

(c) 196 (d) 197

Numerical Value Type

10. In ΔABC, if BC is unity, $\sin\frac{A}{2} = x_1, \sin\frac{B}{2} = x_2, \cos\frac{A}{2} = x_3$ and $\cos\frac{B}{2} = x_4$ with $\left(\frac{x_1}{x_2}\right)^{2007} - \left(\frac{x_3}{x_4}\right)^{2006} = 0$, then the length of AC is _____ .

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11. A triangle has circumradius R and sides a, b, c with $R(a+b)=c\sqrt{ab}$, then the value of $\frac{\sin B}{\sin A}+\sin C$ is _____.

12. The number of triangles, which have points (8, 9) (8, 16) and (20, 25) as the feet of perpendiculars drawn from the vertices on the opposite side, is _____.

13. Let $S_n=\binom{n}{0}-\binom{n-1}{1}+\binom{n-2}{2}-\binom{n-3}{3}+.....$

The series terminates as ${}^pC_q=0$ if $p<q$, then the value of S_{2012} is equal to _____.

14. If $n=12m(m\in N)$ and ${}^nC_0-\frac{{}^nC_2}{(2+\sqrt{3})^2}+\frac{{}^nC_4}{(2+\sqrt{3})^4}-\frac{{}^nC_6}{(2+\sqrt{3})^6}+....=(-1)^m\left(\frac{\sqrt{P}}{1+\sqrt{Q}}\right)^n$, then $P-Q$ is _____.

15. Let $y=g(x)$ be the image of $f(x)=x+\sin x$ about the line $x+y=0$. If the area bounded by $y=g(x)$, x-axis, $x=0$ and $x=2\pi$ is A, then A/π^2 is _____.

16. If $6+f''(x)+f'(x)=x^2+f^2(x)$ be the differential equation of a curve and let P be the point of minima of this curve, then the number of tangents which can be drawn from P to the circle $x^2+y^2=4$ is _____.

17. Consider the unit cube $ABCDA'B'C'D'$, and the points M, N, P such that M is the foot of the perpendicular from A to plane $(A'CD)$, N is the foot of the perpendicular from B to the diagonal $A'C$ and P is the symmetric point of D w.r.t. C then area of triangle MNP is _____.

SOLUTIONS

1. (b): r^{th} term of the series

$$T_r=(-1)^{r-1}\cdot{}^nC_r\left(1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{r}\right).$$

Clearly, it can be obtained from

$(-1)^{r-1}.{}^nC_r(1+x+x^2+....+x^{r-1})$

by integrating within the limits 0 to 1.

Let $a_r=(-1)^{r-1}\,{}^nC_r(1+x+x^2+....+x^{r-1})$, then

$$a_r=(-1)^{r-1}\,{}^nC_r\left(\frac{1-x^r}{1-x}\right)$$

$$\sum_{r=1}^{n}a_r=\frac{1}{x-1}\sum_{r=1}^{n}(-1)^r\,{}^nC_r+\frac{1}{(1-x)}\sum_{r=1}^{n}(-1)^r\,{}^nC_r\cdot x^r$$

$$\Rightarrow\quad\sum_{r=1}^{n}a_r=(1-x)^{n-1}\quad\therefore\quad S_n=\int_0^1(1-x)^{n-1}dx=\frac{1}{n}$$

2. (b): The elements which will give sum 1094 are (1, 1093), (5, 1089), ...

Now, taking one element from each bracket, we get

$$\frac{1}{2}\left(\frac{1093-1}{4}+1\right)=\frac{1}{2}(273+1)=137\text{ elements}$$

$\therefore$ Maximum number of elements in A =137

$\therefore$ Sum of digits = 1 + 3 + 7 = 11

3. (b): Equidistant points from focus are symmetric about axis of parabola.

So,tangent at vertex is parallel to the line joining the two points.

$\therefore$ Its slope $=\frac{3-2}{2-3}=-1$

4. (b): We make a chart of how many trailing zeros factorials have:

Factorial	Number of trailing zeros
0! – 4!	0
5! – 9!	1
10! – 14!	2
15! – 19!	3
20! – 24!	4
25! – 29!	6

We first look at the case when $n!$ has 1 zero and $(2n!)$ has 3 zeros. If n = 5, 6, 7, $(2n)!$ has only 2 zeros. But for n = 8, 9, $(2n)!$ has 3 zeros. Thus, n = 8 and n = 9 work. Secondly, we look at the case when $n!$ has 2 zeros and $(2n)!$ has 6 zeros. If n = 10, 11, 12, $(2n)!$ has only 4 zeros. But for n = 13, 14, $(2n)!$ has 6 zeros. Thus, the smallest four values of n that work are n = 8, 9, 13, 14 which sum to 44. The sum of the digits of 44 is 8 .

5. (a): Let $[x]=k$, where k is an integer such that $1\le k\le 20$. Then we can rewrite $f(x)$ as $k(21^{x-k}-1)$. Now, for this to be less than or equal to 1, we should have $x\le k+\log_{21}\left(\frac{k+1}{k}\right)$.

Combining this with the fact that $[x]=k$, gives $x\in\left[k,k+\log_{21}\left(\frac{k+1}{k}\right)\right]$. So the length of the interval is $\log_{21}\left(\frac{k+1}{k}\right)$. But we want the sum of lengths of all possible intervals such that the inequality holds true. Since all of these intervals must be disjoint, we can sum from k = 1 to k = 20 to get the desired sum .

6. (c): Rewrite $\cos4x-1$ as $2\cos^2 2x-2$.

Now let $a=\cos2x$ and $b=\cos\left(\frac{20\pi^2}{x}\right)$, we get $ab=1$.

Notice that either $a=1$ and $b=1$ or $a=-1$ and $b=-1$. Possible values of $x=k\pi$ where k = 1, 2, 5, 10.

7. (b): All the desired terms are in the form $a^xb^yc^zd^w1^t$, where $x+y+z+w+t=N$. Since the values of x, y, z and w must be at least 1 (t can be 0), let $x'=x-1$, $y'=y-1$, $z'=z-1$ and $w'=w-1$. Now, $x'+y'+$

$z' + w' + t = N - 4$, where $x', y', z', w', t \geq 0$, there are $\binom{(N-4)+4}{4}$ or $\binom{N}{4}$ solutions to this equation. We notice that $1001 = 7 \cdot 11 \cdot 13$, which leads us to guess that N is around these numbers. We see that $\binom{14}{4} = 1001$, therefore $N = 14$

8. (c) : Putting $r = 0, 1$, we get $k = -5/2$ and $y = 1$.
Now, $(1 + kx + yx^2)\ T = 2 + 3x$

9. (d) : Let $a/c = d/b = x/y$ (such that x and y are integers with no common factor), then $a = hx$, $c = hy$, $d = kx$ and $b = ky$ for some $h, k \in N$.
Now, $a^2 + b^2 + c^2 + d^2$ becomes $(h^2 + k^2)(x^2 + y^2)$, which is a composite number, whose each factor being more than or equal to 2.

10. (1) : In given ΔABC, both $\frac{A}{2}$ and $\frac{B}{2}$ lie strictly between $\left(0, \frac{\pi}{2}\right)$ and $\sin x$ is always increasing in $\left(0, \frac{\pi}{2}\right)$ whereas $\cos x$ is always decreasing in $\left(0, \frac{\pi}{2}\right)$.

Now, if $\frac{A}{2} > \frac{B}{2}$ then $\sin\frac{A}{2} > \sin\frac{B}{2}$ and $\cos\frac{A}{2} < \cos\frac{B}{2}$

or $x_1 > x_2$ and $\frac{1}{x_3} > \frac{1}{x_4}$

So, $x_1^{2007}\ x_4^{2006} = x_2^{2007}\ x_3^{2006}$ is not valid in this case.

Similarly for $\frac{A}{2} < \frac{B}{2}$

Therefore, only possible case is when $\frac{A}{2} = \frac{B}{2}$

$\Rightarrow$ $x_1 = x_2$ and $\frac{1}{x_3} = \frac{1}{x_4}$. Also, in that case ΔABC is isosceles with $\angle ABC = \angle CAB$

$\Rightarrow$ $BC = AC = 1$ unit.

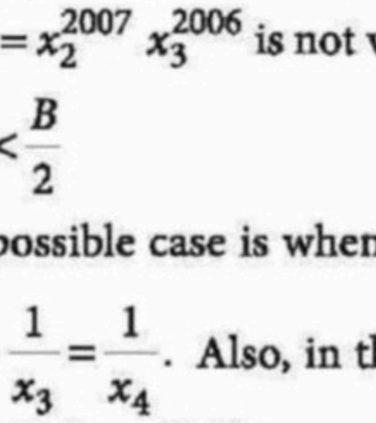
11. (2) : We have, $\frac{(\sin A + \sin B)}{2\sqrt{\sin A \sin B}} = \sin C$

Clearly, L.H.S. ≥ 1 and R.H.S. ≤ 1

(By A.M. - G.M. inequality)

$\Rightarrow$ $\sin A = \sin B$ and $\sin C = 1$

12. (4) : There exist exactly 4 triangle ABC, APC, APB, BPC satisfying condition of which three are obtuse.



13. (0) : From the given series,

$S_n - S_{n-1} + S_{n-2} = 0$ ...(i)

Replacing n by $n - 1$, we get $S_{n-1} - S_{n-2} + S_{n-3} = 0$ ...(ii)

Adding (i) and (ii), we get $S_n + S_{n-3} = 0$ ...(iii)

Replacing n by $n - 3$, we get $S_{n-3} + S_{n-6} = 0$ ...(iv)

Subtracting (iv) from (iii), we get $S_n = S_{n-6}$

Thus, S_n is periodic series of period 6 $\therefore$ $S_{2012} = 0$

14. (5) : Given, $n_{C_0} - \frac{n_{C_2}}{(2+\sqrt{3})^2} + \frac{n_{C_4}}{(2+\sqrt{3})^4} - \frac{n_{C_6}}{(2+\sqrt{3})^6} + \ldots = (-1)^m \left(\frac{\sqrt{P}}{1+\sqrt{Q}}\right)^n$

Clearly, L.H.S. =Real part of $\left(1 + \frac{i}{2+\sqrt{3}}\right)^n$

= Real part of $(1 + i(2 - \sqrt{3}))^n$

=Real part of $\left(1 + i\tan\frac{\pi}{12}\right)^n$

= Real part of $\frac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^n}{\cos^n\frac{\pi}{12}}$

=Real part of $\frac{\left(\cos\frac{n\pi}{12} + i\sin\frac{n\pi}{12}\right)}{\cos^n\frac{\pi}{12}} = \frac{\cos\frac{n\pi}{12}}{\cos^n\frac{\pi}{12}} = \frac{\cos m\pi}{\cos^n\frac{\pi}{12}}$

$= (-1)^m \left(\frac{2\sqrt{2}}{1+\sqrt{3}}\right)^n \quad \left[\because \cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}\right]$

15. (2) : Here, $f(x)$ is an odd function, therefore its graph will be symmetric w.r.t. origin.
Clearly, $g(x) = f^{-1}(x)$

Required area, $A = \int_0^{2\pi} f^{-1}(y)dy = \int_0^{2\pi} f(x)\,dx$

$= \left[[x(f(x))]_0^{2\pi} - \int_0^{2\pi} (1 + \cos x)\,x\,dx\right] = 2\pi^2$

16. (2) : At $P(x, y)$, for minima $f'(x) = 0$ and $f''(x) > 0 \Rightarrow x^2 + f^2(x) - 6 > 0 \Rightarrow x^2 + y^2 > 6$, *i.e.*, P lies outside $x^2 + y^2 = 6$.

$\therefore$ Number of tangents from P to $x^2 + y^2 = 4$ is 2.

17. (0) : We have $\frac{CN}{BC} = \frac{1}{\sqrt{3}} = \frac{A'C}{3}$

and $\frac{DP}{PC} = 2, \frac{CN}{NA'} = \frac{1}{2}$ and $\frac{A'M}{MD} = 1$

Hence by Menelaus's theorem, $\frac{DP}{PC} \cdot \frac{CN}{NA'} \frac{A'M}{MD} = 1$,

i.e., M, N, P are collinear.

OLYMPIAD CORNER

1. We write out all the integers from 1 to 30 inclusive, and cross out some of these so that in the remaining list, no number is the double of any other.
What is the maximum number of integers which can appear in this remaining list?

2. Let $\frac{3}{4} < a < 1$. Prove that the equation
$x^3(x + 1) = (x + a)(2x + a)$ has four distinct real solutions and find these solutions in explicit form.

3. Suppose A, B, C are three non-collinear points corresponding to complex numbers $z_0 = ai$, $z_1 = \frac{1}{2} + bi$, $z_2 = 1 + ci$ (a, b and c being real numbers), respectively. Prove that the curve $z = z_0\cos^4 t + 2z_1\cos^2 t \cdot \sin^2 t + z_2\sin^4 t\ (t \in R)$ shares a single common point with the line bisecting AB and parallel to AC in ΔABC and find this point.

4. Let S be the sum of the elements of the set $\{1, 2, 3, ..., (2p)^n - 1\}$. Let T be the sum of the elements of this set whose representation in base $2p$ consists only of digits from 0 to $p - 1$. Prove that $2^n \times \frac{T}{S} = (p-1)/(2p-1)$.

5. Prove that the average of the numbers $n\sin n°$, $n = 2, 4, 6, ..., 180$ is $\cot 1°$.

6. Let D, E, F be points on the sides BC, CA, AB respectively, of triangle ABC such that $AD \perp BC$, $AF = FB$, and BE is the angle bisector of $\angle B$. Prove that AD, BE, CF are concurrent if and only if, $a^2(a - c) = (b^2 - c^2)(a + c)$, where $a = BC$, $b = CA$, $c = AB$.

7. Two unequal circles of radii R and r touch externally, and P and Q are the points of contact of a common tangent to the circles, respectively. Find the volume of the frustum of a cone generated by rotating PQ about the line joining the centres of the circles.

SOLUTIONS

1. Arrange the numbers 1, 2, 3, ..., 30 into subsets, each of which consists of an odd number and the multiples of that odd number by powers of 2. This gives
{1, 2, 4, 8, 16}
{3, 6, 12, 24}
{5, 10, 20} {7, 14, 28}
{9, 18} {11, 22} {13, 26} {15, 30}
{17} {19} {21} {23} {25} {27} {29}
We can now select numbers from each of these subsets independently of the choice made from any others, and this can be done by choosing as many numbers as possible from each of the sets separately. For example,
choose 1, 4, 16 from the first set {1, 2, 4, 8, 16}
3, 12 from the second set {3, 6, 12, 24}
two each from {5, 10, 20} and {7, 14, 28}
and one each from the remaining 11 sets.
This gives a total of 3 + 2 + 2 + 2 + 11 = 20 numbers remaining.

Generalisation

By arranging any set of n numbers in the fashion suggested in the solution above

1	2	4	8	16	32	64 ....
3	6	12	24	48	96	172 ....
5	10	20	40	80	160	320 ....
7	14	28	56	112	224	448 ....
9	18	36	72	144	288	576 ....
⋮	⋮	⋮	⋮	⋮	⋮	⋮

We can see that all the numbers up to end including n occur just once, any entry in the second column is the double of an entry in the first, any entry in the fourth column is the double of an entry in the third, and so on. So, to build up the largest set of numbers, we need only to choose the numbers from the first, third, fifth column and so on until we have all numbers $\le n$.
This means we need count the numbers in the following columns $\le n$

Col 0	Col 1	Col 2	Col 3	....
1	4	16	64	....
3	12	48	172	....
5	20	80	320	....
7	28	112	448	....
9	36	144	576	....
⋮	⋮	⋮	⋮	

If this count is F_n and C_k is the number of numbers in Col k, which are $\le n$, (C_k = the number of odd multiples of 2^{2k} which are $\le n$). Then

$F_n = C_0 + C_1 + C_2 + ...$

(The series terminates with zero terms after a finite number of steps. The last non-zero term is $C_{[\log_4 n]}$.)

2. Look at the given equation as a quadratic equation in a: $a^2 + 3xa + 2x^2 - x^3 - x^4 = 0$

The discriminant of this equation is

$9x^2 - 8x^2 + 4x^3 + 4x^4 = (x + 2x^2)^2$

Thus, $a = \dfrac{-3x \pm (x+2x^2)}{2}$

The first choice $a = -x + x^2$ yields the quadratic equation $x^2 - x - a = 0$, whose solutions are $x = \dfrac{(1 \pm \sqrt{1+4a})}{2}$.

The second choice $a = -2x - x^2$ yields the quadratic equation $x^2 + 2x + a = 0$, whose solutions are $-1 \pm \sqrt{1-a}$.

The inequalities

$$-1-\sqrt{1-a} < -1+\sqrt{1-a} < \frac{1-\sqrt{1+4a}}{2} < \frac{1+\sqrt{1+4a}}{2}$$

show that the four solutions are distinct.

Indeed $-1+\sqrt{1-a} < \dfrac{1-\sqrt{1+4a}}{2}$ reduces to $2\sqrt{1-a} < 3-\sqrt{1+4a}$, which is equivalent to $6\sqrt{1+4a} < 6+8a$ or $3a < 4a^2$, which is evident.

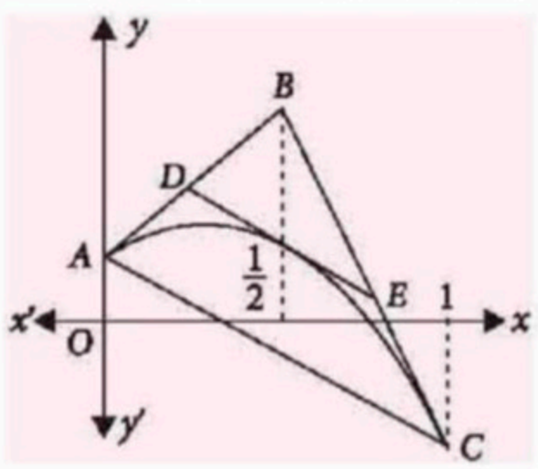
3. Let $z = x + yi$ $(x, y \in R)$, then

$$x + yi = a\cos^4 t \cdot i + 2\left(\frac{1}{2} + bi\right)\cos^2 t \cdot \sin^2 t + (1 + ci)\sin^4 t$$

Separating real and imaginary parts, we get

$x = \cos^2 t \cdot \sin^2 t + \sin^4 t = \sin^2 t,$

$y = a(1-x)^2 + 2b(1-x)x + cx^2 \ (0 \le x \le 1)$



That is,

$y = (a + c - 2b)x^2 + 2(b - a)x + a \ (0 \le x \le 1)$ ...(i)

Since A, B, C are non-collinear, $a + c - 2b \neq 0$.

So, equation (i) is the segment of a parabola (see the diagram). Furthermore, the mid points of AB and BC are $D\left(\dfrac{1}{4}, \dfrac{a+b}{2}\right)$ and $E\left(\dfrac{3}{4}, \dfrac{b+c}{2}\right)$, respectively.

So, the equation of line DE is

$$y = (c-a)x + \frac{1}{4}(3a + 2b - c) \quad ..(ii)$$

Solving equations (i) and (ii) simultaneously, we get $(a+c-2b)\left(x-\dfrac{1}{2}\right)^2 = 0$. Then $x = \dfrac{1}{2}$, since, $a + c - 2b \neq 0$. So, the parabola and line DE have one and only one common point $P\left(\dfrac{1}{2}, \dfrac{a+c+2b}{4}\right)$.

Notice that $\dfrac{1}{4} < \dfrac{1}{2} < \dfrac{3}{4}$, so point P is on the segment DE and satisfies equation (i), as required.

4. We have, $S = \dfrac{[(2p)^n - 1](2p)^n}{2}$. Let R denote the set of numbers that have at most n digits in base $2p$ and contain only the digits from 0 to $p-1$. Thus T is the sum of the elements of R. For example, when $p = 2$ and $n = 3$, we have $R = \{0, 1, 10, 11, 100, 101, 110, 111\}$ and $T = 1110_4 = 84$. Note R will have p^n elements, because each of the n digits (including leading zeros) can be any one of p different numbers. Since each number in $\{0, 1, ..., p-1\}$ appears the same number of times as a digit in R, there will be exactly p^{n-1} elements in R that have k as their t^{th} digit, for $k = 0, 1, ..., p-1$ and $t = 1, 2, ..., n$. Thus,

$$T = \sum_{i=0}^{n-1} (2p)^i \cdot [p^{n-1} \times 0 + p^{n-1} \times 1 + ... + p^{n-1} \times (p-1)]$$

$$= \sum_{i=0}^{n-1} (2p)^i \cdot p^{n-1} \cdot \frac{p(p-1)}{2} = \frac{p^n(p-1)}{2} \cdot \sum_{i=0}^{n-1} (2p)^i$$

$$= \frac{p^n(p-1)}{2} \cdot \frac{(2p)^n - 1}{2}$$

Monthly Test Drive CLASS XI **ANSWER KEY**

1. (a)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (d)
6. (b)	**7.** (b, d)	**8.** (b, c)	**9.** (a, b, c)	**10.** (c)
11. (a, c)	**12.** (a, c, d)	**13.** (b)	**14.** (c)	**15.** (a)
16. (b)	**17.** (6)	**18.** (32)	**19.** (2)	**20.** (1.25)

Hence, $2^n \times \frac{T}{S} = \frac{2^n \cdot \frac{p^n(p-1)}{2} \cdot \frac{(2p)^n - 1}{2p-1}}{\frac{[(2p)^n - 1](2p)^n}{2}} = \frac{p-1}{2p-1}$

as required.

5. Let $x = 2\sin 2° + 4\sin 4° + ... + 90\sin 90° + ... + 178\sin 178°$

$= (2 + 178)\sin 2° + (4 + 176)\sin 4° + ...$

$= 180(\sin 2° + \sin 4° + ... + \sin 88°) + 90\sin 90°$

Then, $\bar{x} = \frac{x}{90} = 2\sin 2° + 2\sin 4° + ... + 2\sin 88° + 1$

$\bar{x}\sin 1° = 2\sin 2°\sin 1° + 2\sin 4°\sin 1° + ... + 2\sin 88°\sin 1° + \sin 1°$

Now, $2\sin 2°\sin 1° = \cos 1° - \cos 3°$

$2\sin 4°\sin 1° = \cos 3° - \cos 5°$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$2\sin 88°\sin 1° = \cos 87° - \cos 89°$

Hence, $\bar{x}\sin 1° = \cos 1° - \cos 89° + \sin 1° = \cos 1°$

Thus, $\bar{x} = \cot 1°$, as required.

6. By Ceva's Theorem, the cevians AD, BE, CF in ΔABC are concurrent if and only if (using directed line segments)

$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$

In this problem, $\frac{AF}{FB} = 1$, and $\frac{CE}{EA} = \frac{a}{c}$. Thus AD, BE, CF are concurrent if and only if $\frac{BD}{DC} = \frac{c}{a}$.

This in turn is true if and only if

$BD = \frac{ac}{a+c}$ and $CD = \frac{a^2}{a+c}$.

Because $AB^2 - BD^2 = BD^2 = AC^2 - CD^2$, these last conditions hold exactly when the following equations are true:

$$AB^2 - \left(\frac{ac}{a+c}\right)^2 = AC^2 - \left(\frac{a^2}{a+c}\right)^2$$

$\Rightarrow (a+c)^2c^2 - a^2c^2 = (a+c)^2b^2 - a^4$

$\Rightarrow a^4 - a^2c^2 = (b^2 - c^2)(a+c)^2;\ a^2(a-c) = (b^2 - c^2)(a+c)$

Therefore, the three lines are concurrent if and only if the given equation holds, as desired.

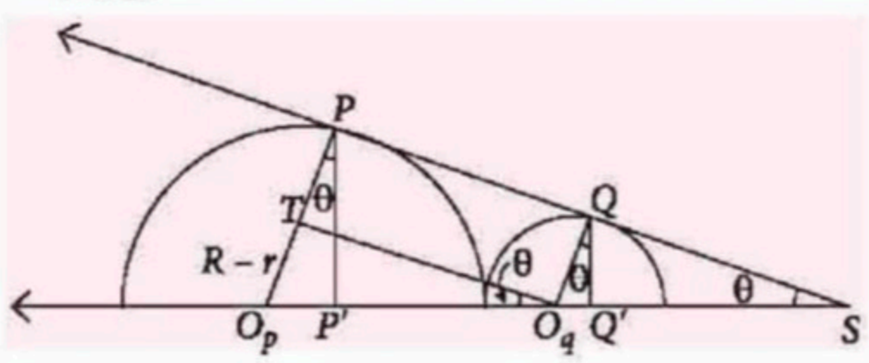
Alternatively, applying the law of cosines gives

$$\frac{BD}{DC} = \frac{c\cos B}{b\cos C} = \frac{c}{b} \cdot \frac{a^2 + c^2 - b^2}{2ac} \cdot \frac{2ab}{a^2 + b^2 - c^2}$$

$$= \frac{a^2 + c^2 - b^2}{a^2 + b^2 - c^2}$$

Again, this equals $\frac{c}{a}$ exactly when the given equation holds.

7. First, we note that triangle ΔTO_pO_q has a right angle angle at T, with $O_pO_q = R + r$ and $O_pT = R - r$. Hence $O_qT = 2\sqrt{Rr}$.



Because of parallel and perpendicular lines, all of angles $\angle PSO_p$, $\angle TO_qO_p$, $\angle O_pPP'$ and $\angle O_qQQ'$ are equal. We denote the common value by θ. From triangle ΔO_pO_qT, we note that $\sin\theta = \frac{R-r}{R+r}$ and $\cos\theta = \frac{2\sqrt{Rr}}{R+r}$

Using various right triangles, we get

$O_pP' = R\sin\theta = \frac{R(R-r)}{R+r}$, $PP' = R\cos\theta = \frac{2R\sqrt{Rr}}{R+r}$

$P'S = PP'\cot\theta = \frac{2R\sqrt{Rr}}{R+r}\frac{2\sqrt{Rr}}{R-r} = \frac{4rR^2}{R^2 - r^2}$,

$Q_qQ' = r\sin\theta = \frac{r(R-r)}{R+r}$

$QQ' = r\cos\theta = \frac{2r\sqrt{Rr}}{R+r}$

$Q'S = QQ'\cot\theta = \frac{2r\sqrt{Rr}}{R+r}\frac{2\sqrt{Rr}}{R-r} = \frac{4r^2R}{R^2 - r^2}$

Hence, $V = \frac{\pi}{3}\left[(PP')^2P'S - (QQ')^2Q'S\right]$

$= \frac{\pi}{3}\left[\frac{4R^3r}{(R+r)^2}\frac{4rR^2}{R^2 - r^2} - \frac{4Rr^3}{(R+r)^2}\frac{4r^2R}{R^2 - r^2}\right]$

$= \frac{16\pi R^2r^2(R^3 - r^3)}{3(R+r)^2(R^2 - r^2)} = \frac{16\pi R^2r^2(R^2 + Rr + r^2)}{3(R+r)^3}$, which is the required volume.

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MOCK TEST PAPER

ISI

Indian Statistical Institute

Exam on 10th May, 2020

*ALOK KUMAR

PART-A

1. The number of sets of two or more consecutive positive integers whose sum is 100 is

(a) 1 (b) 2 (c) 3 (d) 4

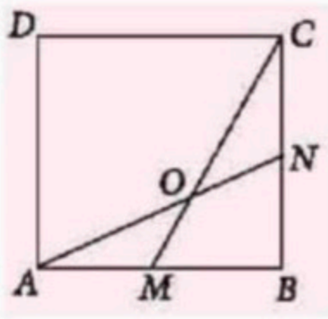
2. The volume of a pyramid whose base is an equilateral triangle of side length 6 and whose other edges are each of length $\sqrt{15}$, is

(a) 9 (b) $\frac{9}{2}$ (c) $\frac{27}{2}$ (d) $\frac{9\sqrt{3}}{2}$

3. A fair die is rolled six times. The probability of rolling at least a five at least five times is

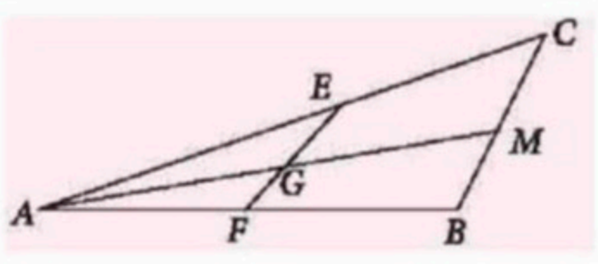
(a) $\frac{13}{729}$ (b) $\frac{12}{729}$ (c) $\frac{2}{729}$ (d) $\frac{3}{729}$

4. In the adjoining figure AB and BC are adjacent sides of square $ABCD$; M is the midpoint of AB; N is the midpoint of BC; and AN and CM intersect at O. The ratio of the area of $AOCD$ to the area of $ABCD$ is



(a) $\frac{5}{6}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{\sqrt{3}}{2}$

5. In triangle ABC shown in the adjoining figure, M is the midpoint of side BC, $AB = 12$ and $AC = 16$. Points E and F are taken on AC and AB, respectively, and lines EF and AM intersect at G. If $AE = 2AF$ then $\frac{EG}{GF}$ equals

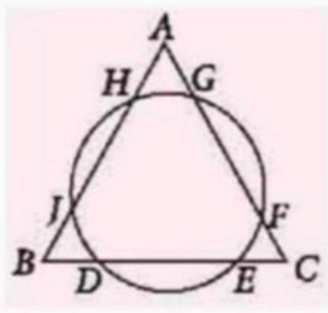


(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{6}{5}$

6. If the value of $\int_0^\infty \frac{\log_e x}{x^2+a^2}dx\,(a>0)$ is $k\frac{\pi \ln a}{8a}$, then the integer part of k is

(a) 4 (b) 6 (c) 3 (d) 1

7. In the adjoining figure, the circle meets the sides of an equilateral triangle at six points. If $AG = 2$, $GF = 13$, $FC = 1$ and $HJ = 7$. If $DE = m\sqrt{n}$, m and n being positive integers, then $n - m^4$ equals



(a) 4 (b) 6 (c) 3 (d) 1

8. Let A be a non-empty set of real numbers and $f: A \to A$ be such that $f(f(x)) = x$ for all x in A. Then f is

(a) a bijection
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

9. If $f(x) = \lim_{t\to\infty} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}$, then range of $f(x)$ is

(a) $\{-1, 1\}$ (b) $\{0, 1\}$
(c) $[-1, 1]$ (d) $\{-1, 0, 1\}$

10. Let $f(x, y)$ be a periodic function satisfying $f(x, y) = f(2x + 2y, 2y - 2x)$ for all $x, y \in R$. Define $g(x) = f(2^x, 0)$, then

(a) $g(x)$ is non-periodic
(b) $g(x)$ is periodic function without fundamental period.
(c) $g(x)$ is periodic function with period 6.
(d) $g(x) = g(x + 12)$

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11. Let $f(x) = x^3 + ax^2 + 6x + \cos 3x$, then f^{-1} will exist if
(a) $a < -3$ (b) $-3 < a < 3$
(c) $a > 3$ (d) $4 < a < 5$

12. A variable circle C has the equation $x^2 + y^2 - 2(t^2 - 3t + 1)x - 2(t^2 + 2t)y + t = 0$, where t is a parameter. The locus of the centre of the circle is
(a) a parabola (b) an ellipse
(c) a hyperbola (d) pair of straight lines

13. An isoceles triangle with base 24 and legs 15 each is inscribed in a circle with centre at (–1, 1), then locus of the centroid of that Δ is
(a) $4(x^2 + y^2) + 8x - 8y - 73 = 0$
(b) $2(x^2 + y^2) + 4x - 4y - 31 = 0$
(c) $2(x^2 + y^2) + 4x - 4y - 21 = 0$
(d) $4(x^2 + y^2) + 8x - 8y - 161 = 0$

14. Consider one side AB of a square $ABCD$ on the line $y = 2x - 17$ and the other two vertices C, D lies on the parabola $y = x^2$ and the maximum possible area of the square $ABCD$ (in square units) is
(a) 160 (b) 1280 (c) 640 (d) 320

15. If $f(x) = \frac{3}{3-x}$ and $g(x) = f(f(x))$, then point of discontinuity of $g(x)$ are α and β, where $\alpha < \beta$. A point $P(t, 2t + 1)$ is any point in x-y plane which lies in opposite side of line $x + y - 1 = 0$ w.r.t. (α, β), then
(a) $t \in R^+$ (b) $t \in R^-$
(c) $t \in (-\infty, \infty)$ (d) none of these

16. The differential equation of all parabolas with axis parallel to the y-axis is
(a) $\frac{d^2y}{dx^2} = 2y + x$ (b) $\frac{d^3y}{dx^3} = 2y$
(c) $\left(\frac{d^2y}{dx^2}\right)^3 = \frac{dy}{dx}$ (d) $\frac{d^3y}{dx^3} = 0$

17. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{xy(x^2 \sin y^2 + 1)}$ is (c being an arbitrary constant)
(a) $x^2(\cos y^2 - \sin y^2 - 2ce^{-y^2}) = 2$
(b) $x^2(\cos y^2 - \sin y^2 - 2e^{-y^2}) = 4c$
(c) $y^2(\cos x^2 - \sin x^2 - 2ce^{-y^2}) = 2$
(d) $y^2(\cos x^2 - \sin x^2 - ce^{-y^2}) = 4c$

18. If A and B are two square matrices such that $AB = B$ and $BA = A$ then for $n \in N$, $(A + B)^n =$
(a) $2^n(A + B)$ (b) $2^{n-1}(A + B)$
(c) $2^{n+1}(A + B)$ (d) $2^{\frac{n}{2}}(A + B)$

19. Let $S = \left\{(a, b) \text{ such that } A^3 = A, \text{where } A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ a & b \end{bmatrix}\right\}$, then $n(S) =$
(a) 1 (b) 2 (c) 3 (d) 0

20. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB then the cosine of angle α is given by
(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

21. The length of the edge of the regular tetrahedron $ABCD$ is 'a'. Points E and F are taken on the edges AD and BD respectively such that 'E' divides $\overrightarrow{DA}$ and 'F' divides $\overrightarrow{BD}$ in the ratio 2 : 1 each. Then area of triangle CEF is
(a) $\frac{5a}{12\sqrt{3}}$ sq. units (b) $\frac{a}{12\sqrt{3}}$ sq. units
(c) $\frac{a^2}{12\sqrt{3}}$ sq. units (d) $\frac{5a^2}{12\sqrt{3}}$ sq. units

22. Let $\vec{r} = (\vec{a} \times \vec{b}) \sin x + (\vec{b} \times \vec{c}) \cos y + 2(\vec{c} \times \vec{a})$ where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors. If $\vec{r}$ is perpendicular to $\vec{a} + \vec{b} + \vec{c}$, then minimum value of $x^2 + y^2$ is
(a) π^2 (b) $\frac{\pi^2}{4}$ (c) $\frac{5\pi^2}{4}$ (d) none of these

23. A circle of radius 1 unit touches positive x-axis and positive y-axis at A and B respectively. A variable line passing through origin intersects the circle in two points D and E. If the area of the triangle DEB is maximum when the slope of the line is m then m^{-2} equals
(a) 4 (b) 6 (c) 3 (d) 1

24. Let the number of integral points on and inside the region bounded by $|x| \le 10$, $|y| \le 10$ and $|x + y| \le 10$ be n. (A point (x, y) is an integral point if x and y are integers). The last digit of n is
(a) 4 (b) 6 (c) 3 (d) 1

PART-B

1. Prove the identities given below :
(i) $\sin 3A \sin^3 (B - C) + \sin 3B \sin^3 (C - A) + \sin 3C \sin^3 (A - B) = 0$
(ii) $\sin 3A \cos^3 (B - C) + \sin 3B \cos^3 (C - A) + \sin 3C \cos^3 (A - B) = \sin 3A \sin 3B \sin 3C$, if $A + B + C = \pi$

2. Let $\frac{a^k}{(a-b)(a-c)}+\frac{b^k}{(b-a)(b-c)}+\frac{c^k}{(c-a)(c-b)}=S_k$.

Prove that

$S_0 = S_1 = 0$, $S_2 = 1$, $S_3 = a + b + c$,

$S_4 = ab + ac + bc + a^2 + b^2 + c^2$,

$S_5 = a^3 + b^3 + c^3 + a^2b + b^2a + c^2a + a^2c + b^2c + c^2b + abc$.

3. Let

$$\sigma_m = a^m\frac{(a+b)(a+c)}{(a-b)(a-c)}+b^m\frac{(b+c)(b+a)}{(b-c)(b-a)}+c^m\frac{(c+a)(c+b)}{(c-a)(c-b)}.$$

Compute σ_1, σ_2, σ_3 and σ_4.

4. Let $2s = a + b + c$ and $2\sigma^2 = a^2 + b^2 + c^2$.
Show that $(\sigma^2 - a^2)(\sigma^2 - b^2) + (\sigma^2 - b^2)(\sigma^2 - c^2) + (\sigma^2 - c^2)(\sigma^2 - a^2) = 4s(s - a)(s - b)(s - c)$.

5. Let n be a positive integer. Find the number of polynomials $P(x)$ with coefficients in {0, 1, 2, 3} such that $P(2) = n$.

6. At a math camp, all m students share exactly one common friend, $m \geq 3$. (If A is a friend of B, then B is a friend of A. Also, A is not his own friend.) Suppose person P has the largest number of friends. Determine what that number is.

7. Let $A_1 A_2 \ldots A_{12}$ be a regular dodecagon with O as its center. Triangular regions OA_iA_{i+1}, $1 \leq i \leq 12$ (and $A_{13} = A_1$) are to be colored red, blue, green, or yellow such that adjacent regions are colored in different colors. In how many ways can this be done ?

8. (i) Determine the set of positive integers n for which 3^{n+1} divides $2^{3^n} + 1$.

(ii) Prove that 3^{n+2} does not divide $2^{3^n} + 1$ for any positive integer n.

9. For a positive integer n, define $A(n)$ to be $\frac{(2n)!}{(n!)^2}$

Determine the sets of positive integers n for which
(i) $A(n)$ is an even number;
(ii) $A(n)$ is a multiple of 4.

10. Given a triangle ABC in a plane Σ find the set of all points P lying in the plane Σ such that the circumcircles of triangles ABP, BCP and CAP are congruent.

11. Suppose P is an interior point of a triangle ABC and AP, BP, CP meet the opposite sides BC, CA, AB in D, E, F respectively. Show that $\frac{AF}{FB}+\frac{AE}{EC}=\frac{AP}{PD}$.

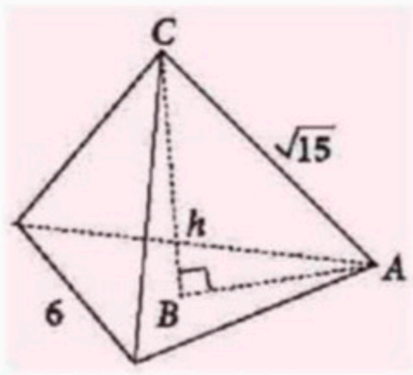
SOLUTIONS

PART-A

1. (b) : The number of consecutive integers in a set is either odd or even. If odd, let their number be $2n + 1$ and their average be x, the middle integer. Then $(2n + 1)\,x = 100$ and $x=\frac{100}{(2n+1)}$, so that $2n + 1$ can only be 5 or 25. If $2n + 1 = 5$, then $x = 20$ and $n = 2$, so that the integers are 18, 19, 20, 21, 22. If $2n + 1 = 25$, $x = 4$ and $n = 12$, which is impossible because the integers must be positive.

If the number of consecutive integers is an even number $2n$, let the average of the integers (half way between the middle pair) be denoted by x. Then $2nx = 100$, $x=\frac{50}{n}$. Since x is a half integer, so $2x=\frac{100}{n}$ is an integer, but $x=\frac{50}{n}$ is not an integer, so that n is 4, 20 or 100. For $n = 4$, $x=12\frac{1}{2}$ and the integers are 9, 10, ..., 16; $n = 20$ and 100 are impossible since the integers are positive. Hence there are exactly two sets of positive integers whose sum is 100.

2. (a) : The volume of a pyramid is equal to one third the area of the base times the altitude. The base of the given pyramid is an equilateral triangle with sides of length 6; hence it has area $9\sqrt{3}$. Altitude h of the given pyramid may be found by applying the Pythagoras theorem to right ΔABC in the adjoining diagram.



Here B is the center of the base, so that $AB=\frac{2}{3}(3\sqrt{3})=2\sqrt{3}$,

$h^2=(\sqrt{15})^2-(2\sqrt{3})^2$

$\Rightarrow h=\sqrt{3}$. Therefore, the volume of the pyramid is $\frac{1}{3}(9\sqrt{3})(\sqrt{3})=9$.

3. (a) : Let A be the event of rolling at least a five; then the probability of A is $\frac{2}{6}=\frac{1}{3}$. In six rolls of a die the probability of event A happening six times is $\left(\frac{1}{3}\right)^6$. The probability of getting exactly five successes and one failure of A in a specific order is $\left(\frac{1}{3}\right)^5.\frac{2}{3}$. Since there are six ways to do this, the probability of getting five successes and one failure of A in any order is

$$6\left(\frac{1}{3}\right)^5 \cdot \frac{2}{3} = \frac{12}{729}$$

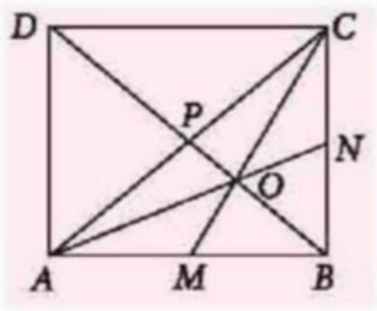
$\therefore$ The probability of getting all successes or five successes and one failure in any order is $\frac{12}{729} + \left(\frac{1}{3}\right)^6 = \frac{13}{729}$

4. (c) : In the adjoining figure, diagonals AC and DB are drawn. Since O is the intersection of the medians of ΔABC, the altitude of ΔAOB from O is $\frac{1}{3}$ the altitude of ΔABC from C; *i.e.* $\frac{1}{3}$ the side length, s, of the square.

Hence, area of ΔAOB

$= \frac{1}{3}$ (area of ΔABC)

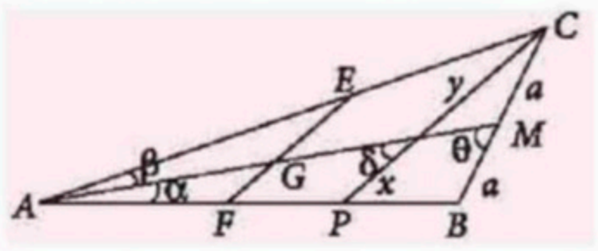
$= \frac{1}{3}\left(\frac{1}{2}s^2\right) = \frac{1}{6}s^2.$



Similarly, area $\Delta COB = \frac{1}{6}s^2$. The area of $AOCD$ is obtained by subtracting the areas of triangles AOB and COB from that of the square, so area

$$AOCD = s^2 - \frac{1}{3}s^2 = \frac{2}{3}s^2.$$

5. (a) : Construct line CP parallel to EF and intersecting AB at P. Then, $\frac{AP}{AC} = \frac{AF}{AE} \Rightarrow \frac{AP}{16} = \frac{AF}{2AF} \Rightarrow AP = 8.$



Let a, x, y, α, β, δ and θ be as shown in the adjoining diagram. The desired ratio $\frac{EG}{GF}$ is the same as $\frac{y}{x}$ which we now determine. By the law of sines,

$$\frac{a}{\sin\alpha} = \frac{12}{\sin\theta}; \quad \frac{a}{\sin\beta} = \frac{16}{\sin(180^\circ - \theta)} = \frac{16}{\sin\theta}$$

Hence, $\frac{\sin\beta}{\sin\alpha} = \frac{3}{4}$.

Moreover $\frac{x}{\sin\alpha} = \frac{8}{\sin\delta}; \frac{y}{\sin\beta} = \frac{16}{\sin(180^\circ - \delta)} = \frac{16}{\sin\delta}$

Hence, $\frac{y}{x} = 2\frac{\sin\beta}{\sin\alpha} = \frac{3}{2}$

6. (a) : $\int_0^\infty \frac{\log_e x}{x^2 + a^2}dx = \underbrace{\int_0^a \frac{\ln x}{a^2 + x^2}dx}_{I_1} + \underbrace{\int_a^\infty \frac{\ln x}{a^2 + x^2}dx}_{I_2}$

Now, substitute $x = \frac{a^2}{y}$ in I_2

$$I_2 = -\int_a^0 \frac{2\ln a - \ln y}{a^2 + y^2}dy = \int_0^a \frac{2\ln a - \ln y}{a^2 + y^2}dy$$

$$\therefore \quad I = I_1 + I_2 = 2\ln a \int_0^a \frac{dx}{a^2 + x^2}$$

$$= 2\ln a\left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]_0^a = \frac{\pi \ln a}{2a}$$

7. (b) : Let $BD = x$, $EC = y$, $AH = z$

Now, $AG \times AF = AH \times AJ$

$\Rightarrow \quad 2 \times 15 = z \times (z + 7) \Rightarrow z^2 + 7z - 30 = 0$

$\Rightarrow \quad z = 3$

Now $AC = AB \Rightarrow 16 = 3 + 7 + BJ \Rightarrow BJ = 6$

Now $BJ \times BH = BD \times BE$

So, $6 \times 13 = x \times (16 - y)$ ...(1)

Also $CE \times CD = CF \times CG$

$\Rightarrow \quad y(16 - x) = 1 \times 14$ ...(2)

Solving (1) and (2), we get $x = 10 - \sqrt{22}, y = 6 - \sqrt{22}$

8. (a) : Let $x, y \in A$ and $f(x) = f(y)$

$\Rightarrow \quad f(f(x)) = f(f(y)) \Rightarrow x = y$ so f is one - one

Also $a \in A \Rightarrow f(f(a)) = a \Rightarrow f(b) = a$ where $b = f(a) \in A$.

So f is onto.

9. (d) : $f(x) = \lim_{t\to\infty} \frac{(1 + \sin\pi x)^t - 1}{(1 + \sin\pi x)^t + 1}$

$$= \begin{cases} \dfrac{1 - \dfrac{1}{(1+\sin\pi x)^t}}{1 + \left(\dfrac{1}{1+\sin\pi x}\right)^t}, & \sin\pi x > 0 \\ \dfrac{0-1}{0+1}, & \sin\pi x < 0 \\ \dfrac{1-1}{1+1}, & \sin\pi x = 0 \end{cases}$$

$$= \begin{cases} 1, & \sin\pi x > 0 \\ -1, & \sin\pi x < 0 \\ 0, & \sin\pi x = 0 \end{cases}$$

$\therefore$ Range of $f = \{-1, 0, 1\}$

10. (d) : $f(x, y) = f(2x + 2y, 2y - 2x)$

$= f(2(2x + 2y) + 2(2y - 2x), 2(2y - 2x) - 2(2x + 2y))$

$= f(8y, -8x) = f(8(-8x), -8(8y)) = f(-64x, -64y)$

$= f(64 \times 64x, 64 \times 64y) = f(2^{12}x, 2^{12}y)$

$\therefore \quad f(2^{12}x, 0) = f(x, 0)$

$\therefore \quad g(x) = f(2^x, 0) = f(2^{x+12}, 0) = g(x + 12)$

$\therefore$ g is periodic with period 12

$\therefore$ $g(x) = g(x + 12)$

11. (b) : Given, $f(x) = x^3 + ax^2 + 6x + \cos 3x$ ...(1)

$\therefore$ $f'(x) = 3x^2 + 2ax + 6 - 3\sin 3x$ ...(2)

Clearly $f(x)$ is continuous and $f(-\infty) = -\infty$, $f(\infty) = \infty$

Therefore range of $f = (-\infty, \infty)$ $\therefore$ f is onto.

$\therefore$ For f^{-1} to exist, f must be one-one.

For this $f'(x) \geq 0\ \forall\ x \in R$ [$\because$ coeff. of x^2 in $f'(x)$ is +ve]

But $f'(x) = 3x^2 + 2ax + 3 + 3(1 - \sin 3x) \geq 3x^2 + 2ax + 3$

[$\because$ $-1 \leq \sin 3x \leq 1$]

$\therefore$ f will be necessarily increasing and hence it is also one-one if $3x^2 + 2ax + 3 \geq 0\ \forall\ x \in R$

$\Rightarrow$ $D < 0 \Rightarrow 4a^2 - 36 < 0 \Rightarrow a^2 - 9 < 0 \Rightarrow -3 < a < 3$

12. (a) : Centre is given by $x = t^2 - 3t + 1$ ...(1)

and $y = t^2 + 2t$ ...(2)

(2)–(1) gives $-x + y = 5t - 1 \Rightarrow t = \dfrac{1 - x + y}{5}$

Substituting the value of t in (2), we get

$$y = \left(\frac{y - x + 1}{5}\right)^2 + 2\left(\frac{y - x + 1}{5}\right)$$

$\Rightarrow$ $25y = (y - x + 1)^2 + 10(y - x + 1)$

$\Rightarrow$ $25y = y^2 + x^2 + 1 - 2xy - 2x + 2y + 10y - 10x + 10$

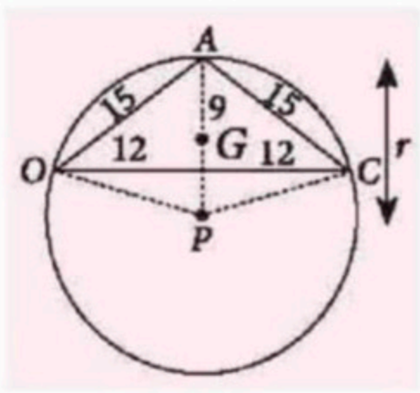
$\Rightarrow$ $x^2 + y^2 - 2xy - 12x - 13y + 11 = 0$,

which is a parabola as $\Delta \neq 0$ and $h^2 = ab$

13. (d) : $(r - 9)^2 + 12^2 = r^2$

$\Rightarrow$ $81 - 18r + 144 = 0$

$\Rightarrow$ $r = \dfrac{225}{18} = \dfrac{25}{2}$



Locus of the centroid of the triangle will be a concentric circle with radius = PG

$$= (r - 9) + \frac{1}{3}(9) = r - 6 = \frac{25}{2} - 6 = \frac{13}{2}$$

$\therefore$ Required locus is $(x+1)^2 + (y-1)^2 = \left(\dfrac{13}{2}\right)^2$

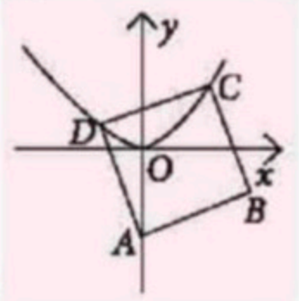
$\Rightarrow$ $x^2 + y^2 + 2x - 2y - \dfrac{161}{4} = 0$

14. (b) : Let equation of AB is $y = 2x - 17$

and that of CD is $y = 2x + c$

where $C = (x_1, y_1)$ and $D = (x_2, y_2)$.

Slope of $CD = \dfrac{y_1 - y_2}{x_1 - x_2} = 2$ ...(1)



Let $CD = a = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow$ $a^2 = 5(x_1 - x_2)^2$ [Using (1)]

$\Rightarrow$ $a^2 = 5[(x_1 + x_2)^2 - 4x_1x_2]$ ...(2)

$\therefore$ $y = x^2$, $y = 2x + c$

$\Rightarrow$ $x^2 - 2x - c = 0$

$\therefore$ $x_1 + x_2 = 2$, $x_1x_2 = -c$

From (2), $a^2 = 5(4 + 4c)$ ...(3)

Let $A = (k, 2k - 17)$ be any point on $y = 2x - 17$

$$\therefore\ AD = a = \frac{|2k - 2k + 17 + c|}{\sqrt{5}} \Rightarrow a^2 = \frac{(17 + c)^2}{5} \quad ...(4)$$

From (4) and (5), $a^2 = 1280$

15. (b) : $f(x) = \dfrac{3}{3 - x}$

$$\therefore\ g(x) = f(f(x)) = \frac{3}{3 - \frac{3}{3 - x}} = \frac{9 - 3x}{9 - 3x - 3} = \frac{3 - x}{2 - x}$$

So, $g(x)$ is discontinuous at $x = 2$ and $x = 3$.

$\Rightarrow$ $\alpha = 2, \beta = 3$ ($\because$ $\alpha < \beta$)

$(t, 2t + 1)$ and $(2, 3)$ lies in opposite side of $x + y - 1 = 0$.

$\Rightarrow$ $(t + 2t)(4) < 0 \Rightarrow t < 0 \Rightarrow t \in R^-$

16. (d) : If (h, k) is the vertex of the parabola, its equation will be $(x - h)^2 = \pm 4a(y - k)$

or this reduces to general form $x^2 + ax + by + c = 0$

Differentiating w.r.t. x, we get $2x + a + by_1 = 0$

Again differentiating w.r.t. x, we get $2 + by_2 = 0$

$\Rightarrow$ $y_2 = \dfrac{-2}{b}$

Again differentiating w.r.t. x, we get $y_3 = 0$; *i.e.*, $\dfrac{d^3y}{dx^3} = 0$

is the required differential equation.

17. (a) : Put $u = y^2$, then equation will be transformed to the linear equation $\dfrac{d}{du}\left(\dfrac{-1}{2x^2}\right) - \dfrac{1}{2x^2} = \dfrac{\sin u}{2}$, I.F. $= e^u$

$\therefore$ Solution is $e^u\left(-\dfrac{1}{2x^2}\right) = \displaystyle\int \frac{e^u \sin u}{2}\, du$

$\Rightarrow$ $x^2(\cos y^2 - \sin y^2 - 2ce^{-y^2}) = 2$

18. (b) : $AB = B$, $BA = A \Rightarrow A^2 = A$, $B^2 = B$

$(A + B)^2 = A^2 + AB + BA + B^2 = A + B + A + B = 2(A + B)$

$(A + B)^3 = 2(A + B)(A + B) = 2 \cdot 2(A + B) = 2^2(A + B)$

$(A + B)^4 = 2^2(A + B)(A + B) = 2^3(A + B)$

$\therefore$ $(A + B)^n = 2^{n-1}(A + B)$

19. (c) : Characterstic polynomial of A is

$$|A - \lambda I| = \begin{vmatrix} \frac{1}{2} - \lambda & \frac{1}{2} \\ a & b - \lambda \end{vmatrix} = 0 = \lambda^2 - p\lambda + q,$$

where $p = b + \dfrac{1}{2}$, $q = \dfrac{b - a}{2}$

According to Cayley hamilton theorem, we have

$A^2 - pA + qI = O$

$\Rightarrow \quad A^3 = pA^2 - qA = p(pA - qI) - qA$

Now, $A^3 = A \Rightarrow (p^2 - q - 1)A = pqI$

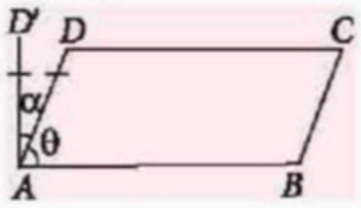
$\Rightarrow \quad p^2 - q - 1 = pq = 0 \Rightarrow q = p^2 - 1$ and $p(p^2 - 1) = 0$

$\Rightarrow \quad (p, q) = (-1, 0), (0, -1), (1, 0)$

Correspondingly, we get

$$(a, b) = \left(\frac{-3}{2}, \frac{-3}{2}\right), \left(\frac{3}{2}, \frac{-1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)$$

20. (b): $\cos\theta = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} = \dfrac{8}{9}$



Now, $\cos^{-1}\dfrac{8}{9} + \alpha = \dfrac{\pi}{2}$ $\quad \left(\because \theta + \alpha = \dfrac{\pi}{2}\right)$

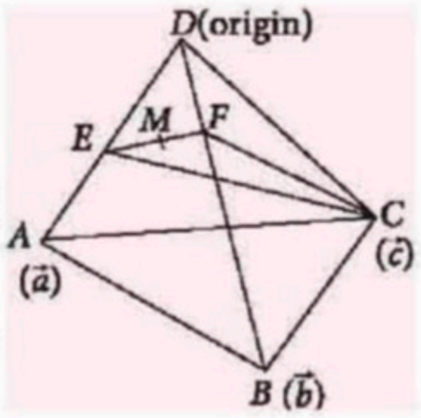
$\Rightarrow \sin\alpha = \dfrac{8}{9} \quad \therefore \cos\alpha = \sqrt{1 - \dfrac{64}{81}} = \dfrac{\sqrt{17}}{9}$

21. (d): We have, $|\vec{a}| = |\vec{b}| = |\vec{c}|$

Also, $|\vec{b} - \vec{a}| = |\vec{b} - \vec{c}| = |\vec{c} - \vec{a}| = a$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{CA}\cdot\overrightarrow{CB} = \overrightarrow{BA}\cdot\overrightarrow{BC} = a^2\cos\dfrac{\pi}{3}$

$\Rightarrow \quad (\overrightarrow{EF})^2 = \dfrac{a^2}{3} \Rightarrow |\overrightarrow{EF}| = \dfrac{a}{\sqrt{3}}$



$|\overrightarrow{CF}| = |\overrightarrow{CE}| = \dfrac{\sqrt{7}a}{3}$ and $|\overrightarrow{CM}| = \dfrac{5a}{6}$

where 'M' is middle point of EF.

Area of $\Delta CEF = (1/2)\,|\overrightarrow{EF}|\,|\overrightarrow{CM}|$

$= \dfrac{1}{2}\times\dfrac{a}{\sqrt{3}}\times\dfrac{5a}{6} = \dfrac{5a^2}{12\sqrt{3}}$ sq.units

22. (c): We have, $\vec{r} = (\vec{a}\times\vec{b})\sin x + (\vec{b}\times\vec{c})\cos y + 2(\vec{c}\times\vec{a})$

Also, $\vec{r}\cdot(\vec{a} + \vec{b} + \vec{c}) = 0$

$\therefore \quad [\vec{a}\ \vec{b}\ \vec{c}](\sin x + \cos y + 2) = 0$

$[\vec{a}\ \vec{b}\ \vec{c}] \neq 0 \Rightarrow \sin x + \cos y = -2$, which is possible only when $\sin x = -1$ and $\cos y = -1$

For $x^2 + y^2$ to be minimum, $x = -\dfrac{\pi}{2}$ and $y = \pi$

$\Rightarrow$ Minimum value of $(x^2 + y^2) = \dfrac{\pi^2}{4} + \pi^2 = \dfrac{5\pi^2}{4}$

23. (c): The equation of the circle is $(x - 1)^2 + (y - 1)^2 = 1$

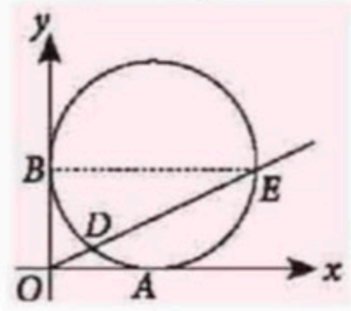
$\Rightarrow \quad x^2 + y^2 - 2x - 2y + 1 = 0$ ....(1)

Let the equation of the variable straight line be

$y = mx$ ....(2)

Solving (1) and (2), we get $(1 + m^2)x^2 - 2x(1 + m) + 1 = 0$

$\therefore$ Length of $DE = \sqrt{\dfrac{8m}{1 + m^2}}$



Area of ΔDEB,

$A = \dfrac{1}{2}DE \times$ distance of B from DE

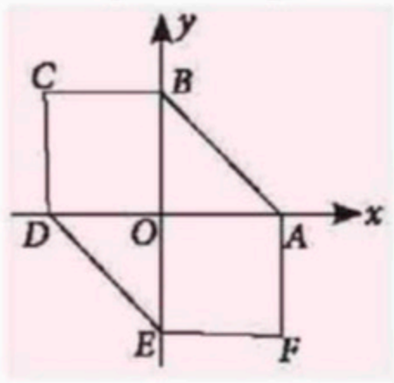
$\therefore \quad A^2 = \dfrac{1}{4}\cdot\dfrac{8m}{1+m^2}\cdot\dfrac{1}{1+m^2} = \dfrac{2m}{(1+m^2)^2} \Rightarrow A = \dfrac{\sqrt{2m}}{1+m^2}$

$\dfrac{dA}{dm} = \dfrac{1 - 3m^2}{\sqrt{2m}(1+m^2)^2} = 0 \Rightarrow m = \pm\dfrac{1}{\sqrt{3}}$

$\dfrac{d^2A}{d^2m} < 0$ if $m = \dfrac{1}{\sqrt{3}}$

$\therefore$ Area is maximum for $m = \dfrac{1}{\sqrt{3}}$.

24. (d): The region is shown in the adjacent figure.



The region $OBCD$ contains $11 \times 11 = 121$ integral points. Similarly the region $OAFE$ contains $11 \times 11 = 121$ integral point. But origin is common so the number of integral points in both the regions $= 121 + 121 - 1 = 241$.

Now the region OAB excluding line segments OA and OB is given by $x + y \leq 10, x > 0, y > 0$.

So, for $x = 1$, y can take 9 values

For $x = 2$, y can take 8 values

$\vdots \quad \vdots$

For $x = 9$, y can take 1 value

$\therefore$ The region OAB contains $1 + 2 + 3 + \ldots + 9$ integral points $= 45$ points.

Similarly, the region ODE contains 45 points.

So, the total number of integral points

$= 241 + 45 + 45 = 331$.

PART-B

1. (i) We have, $\sin 3x = 3\sin x - 4\sin^3 x$

Therefore,

$$\sum \sin 3A \sin^3(B - C) = \frac{1}{4}\sum \sin 3A\{3\sin(B - C) - \sin 3(B - C)\}$$

$$= \frac{3}{4}\sum \sin 3(B + C)\sin(B - C) - \frac{1}{4}\sum \sin 3(B + C)\sin 3(B - C)$$

$$= \frac{3}{8}\sum\{\cos(2B + 4C) - \cos(4B + 2C)\} - \frac{1}{8}\sum(\cos 6C - \cos 6B)$$

$$=\frac{3}{8}\{\cos 2(B+2C)-\cos 2(C+2B)+\cos 2(C+2A)$$
$$-\cos 2(A+2C)+\cos 2(A+2B)-\cos 2(B+2A)\}$$
$$-\frac{1}{8}\{\cos 6C-\cos 6B+\cos 6A-\cos 6C+\cos 6B-\cos 6A\}$$

But $\cos(2B+4C)=\cos(2B+4A)$, $\cos(2C+4B)$
$=\cos(2C+4A)$, $\cos(2A+4C)=\cos(2A+4B)$.

$\therefore \quad \sum \sin 3A \sin^3(B-C)=0$

(ii) Since, $\cos 3x = 4\cos^3 x - 3\cos x$

$\therefore \quad \sum \sin 3A \cos^3(B-C)$

$$=\frac{1}{4}\sum \sin 3(B+C)\{\cos 3(B-C)+3\cos(B-C)\}$$
$$=\frac{1}{4}\sum \sin 3(B+C)\cos 3(B-C)$$
$$+\frac{3}{4}\sum \sin 3(B+C)\cos(B-C)$$
$$=\frac{1}{8}\sum(\sin 6B+\sin 6C)+\frac{3}{8}\sum\{\sin(4B+2C)+\sin(2B+4C)\}$$
$$=\frac{1}{4}(\sin 6A+\sin 6B+\sin 6C)=\sin 3A \sin 3B \sin 3C$$

2. Consider

$$S_0=\frac{1}{(a-b)(a-c)}+\frac{1}{(b-a)(b-c)}+\frac{1}{(c-a)(c-b)}$$

Reducing the fractions to a common denominator, we have

$$S_0=\frac{1}{(a-b)(a-c)(b-c)}[(b-c)-(a-c)+(a-b)]=0$$

$$S_1=\frac{a}{(a-b)(a-c)}+\frac{b}{(b-a)(b-c)}+\frac{c}{(c-a)(c-b)}$$
$$=\frac{1}{(a-b)(a-c)(b-c)}[a(b-c)-b(a-c)+c(a-b)]=0$$

$$S_2=\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-a)(b-c)}+\frac{c^2}{(c-a)(c-b)}$$
$$=\frac{1}{(a-b)(a-c)(b-c)}[a^2(b-c)-b^2(a-c)+c^2(a-b)]$$

Consider the numerator, we have

$a^2(b-c)-b^2(a-c)+c^2(a-b)$
$=ab(a-b)-c(a^2-b^2)+c^2(a-b)$
$=(a-b)(ab-ca-cb+c^2)$
$=(a-b)[a(b-c)-c(b-c)]$
$=(a-b)(b-c)(a-c)$

$\therefore \quad S_2 = 1$. S_3, S_4 and S_5 can be computed analogously, but we shall proceed here in a somewhat different way. It is easily seen that there exists the following identity

$(x-a)(x-b)(x-c)=x^3-(a+b+c)x^2$
$+(ab+ac+bc)\,x-abc.$

Putting, $x=a$, $x=b$ and $x=c$, in turn, we get the following equalities

$a^3-(a+b+c)a^2+(ab+ac+bc)a-abc=0$,
$b^3-(a+b+c)b^2+(ab+ac+bc)b-abc=0$,
$c^3-(a+b+c)c^2+(ab+ac+bc)c-abc=0$.

Further, divide the first of them by $(a-b)(a-c)$, the second by $(b-c)(b-a)$ and the third by $(c-a)(c-b)$, and add them term by term. Then

$S_3-(a+b+c)S_2+(ab+ac+bc)S_1-abc\,S_0=0$.

But we know that $S_0=S_1=0$, $S_2=1$.

$\therefore \quad S_3=a+b+c$.

To compute S_4 let us take the preceding identity and multiply its members by x. We obtain

$x(x-a)(x-b)(x-c)=x^4-(a+b+c)\,x^3$
$+(ab+ac+bc)x^2-abcx.$

Proceeding analogously, we find that

$S_4-(a+b+c)S_3+(ab+ac+bc)S_2-abc\,S_1=0$.

Hence, $S_4=(a+b+c)S_3-(ab+ac+bc)S_2$
$=(a+b+c)^2-ab-ac-bc$
$=a^2+b^2+c^2+ab+ac+bc$.

Likewise, for computing S_5 (multiplying the original identity by x^2), we have

$S_5-(a+b+c)S_4+(ab+ac+bc)S_3-abc\,S_2=0$.

Consequently,

$S_5=(a+b+c)(a^2+b^2+c^2+ab+ac+bc)$
$-(ab+ac+bc)(a+b+c)+abc$
$=(a+b+c)\,(a^2+b^2+c^2)+abc$
$=a^3+b^3+c^3+a^2b+a^2c+b^2a+b^2c+c^2a+c^2b+abc$.

EXAM CORNER 2020

Exam	Date
VITEEE	13th to 19th April
SRMJEEE	12th to 20th April
J & K CET	26th April **(Revised)**
Kerala PET	20th & 21st April
Karnataka CET	22nd & 23rd April
COMEDK (Engg.)	10th May
UPSEE	10th May
AMU (Engg.)	31st May **(Revised)**
BITSAT	16th to 25th May
MHT CET	20th to 23rd April **(Revised)**
JEE Advanced	17th May

3. In the previous question, we have

$$S_m = \frac{a^m}{(a-b)(a-c)} + \frac{b^m}{(b-a)(b-c)} + \frac{c^m}{(c-a)(c-b)}$$

Let us take the first term of our sum σ_m and transform it .

$$a^m \frac{(a+b)(a+c)}{(a-b)(a-c)} = \frac{(a+b+c)a^{m+1} + a^{m-1} \cdot abc}{(a-b)(a-c)}$$

Making use of a circular permutation, we get similar expressions for the second and third terms of σ_m. Now adding all these terms, we get

$\sigma_m = (a + b + c)S_{m+1} + abc\, S_{m-1}$

$\therefore \quad \sigma_1 = (a + b + c)S_2 + abc\, S_0 = a + b + c$

$(\because \quad S_2 = 1, S_0 = 0),$

$\sigma_2 = (a + b + c)S_3 + abc\, S_1 = (a + b + c)^2$

$(\because \quad S_3 = a + b + c, S_1 = 0)$

$\sigma_3 = (a + b + c)S_4 + abc\, S_2 = (a + b + c)(a^2 + b^2 + c^2 + ab + ac + bc) = abc$

$\sigma_4 = (a + b + c)\, S_5 + abc\, S_3 = (a + b + c)$

$[(a + b + c)\, (a^2 + b^2 + c^2) + 2abc]$

4. Consider

$(2\sigma^2 - 2a^2)(2\sigma^2 - 2b^2) = (a^2 + c^2 - b^2)(b^2 + c^2 - a^2)$

$= c^4 - (a^2 - b^2)^2$

Using a circular permutation, we have

$(2\sigma^2 - 2b^2)(2\sigma^2 - 2c^2) = a^4 - (b^2 - c^2)^2,$

$(2\sigma^2 - 2c^2)(2\sigma^2 - 2a^2) = b^4 - (c^2 - a^2)^2.$

Hence, $4[(\sigma^2 - a^2)(\sigma^2 - b^2) + (\sigma^2 - b^2)(\sigma^2 - c^2) + (\sigma^2 - c^2)\,(\sigma^2 - a^2)] = a^4 + b^4 + c^4 - (a^2 - b^2)^2 - (b^2 - c^2)^2 - (c^2 - a^2)^2$

$= -a^4 - b^4 - c^4 + 2a^2b^2 + 2a^2c^2 + 2b^2c^2$

$= -[a^4 - 2(b^2 + c^2)a^2 + (b^2 - c^2)^2]$

$= -[a^4 - 2(b^2 - c^2)a^2 + (b^2 - c^2)^2 - 4a^2c^2]$

$= 4a^2c^2 - (a^2 - b^2 + c^2)^2$

$= (2ac + a^2 - b^2 + c^2)(2ac - a^2 + b^2 - c^2)$

$= (a + b + c)(a + c - b)(b - a + c)(b + a - c).$

But $a + b + c = 2s$ (Given)

$\Rightarrow \quad a + b - c = 2(s - c),\ a + c - b = 2(s - b),$

$b + c - a = 2(s - a)$

and we see that the identity is valid.

5. Let $S = \{0, 1, 2, 3\}$ and let

$P(x) = a_m x^m + a_{m-1}x^{m-1} + \ldots + a_1 x + a_0$, where $a_i \in S$.

Then $P(2) = 2^m a_m + 2^{m-1}a_{m-1} + \ldots + 2a_1 + a_0$.

We are trying to find the number of sequences $(a_0, a_1, \ldots)$ with each $a_i \in S$ such that

$$a_0 + 2a_1 + 4a_2 + \ldots = \sum_{i=0}^{\infty} 2^i a_i = n$$

We consider the generating function

$f(x) = (1 + x + x^2 + x^3)(1 + x^2 + x^4 + x^6)(1 + x^4 + x^8 + x^{12})\ldots,$

where $1 + x + x^2 + x^3$ represents the different choices for a_0, $1 + x^2 + x^4 + x^6$ represents the different choices for a_1, $1 + x^4 + x^8 + x^{12}$ represents the different choices for a_2, and so on. It suffices to find the coefficient of term x^n in $f(x)$.

Note that $f(x) = \frac{x^4-1}{x-1} \cdot \frac{x^8-1}{x^2-1} \cdot \frac{x^{16}-1}{x^4-1} \cdot \frac{x^{64}-1}{x^8-1} \ldots$

$$= \frac{1}{(x-1)(x^2-1)}$$

as each term in the numerator occurs in the denominator of the fraction two terms away. By partial fractions, we obtain

$$f(x) = \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

$$= \frac{-2}{4(x^2-1)} + \frac{1}{2(x-1)^2} = \frac{1}{2}\left((x-1)^{-2} + \frac{1}{1-x^2}\right)$$

Expanding the two functions in the last equation, we find that

$$f(x) = \frac{1}{2}\left[\left(1 - \binom{-2}{1}x + \binom{-2}{2}x^2 - \ldots\right) + (1 + x^2 + x^4 + \ldots)\right]$$

Since $\binom{-2}{n} = \frac{(-2)(-3)\ldots(-2-n+1)}{n!} = (-1)^n(n+1)$, we obtain

$$f(x) = \frac{1}{2}[(1 + 2x + 3x^2 + \ldots) + (1 + x^2 + x^4 + \ldots)]$$

$$= 1 + x + 2x^2 + 2x^3 + 3x^4 + 3x^5 + \ldots$$

$$= \sum_{m=0}^{\infty}\left(\left[\frac{m}{2}\right] + 1\right)x^m$$

Thus, the coefficient of x^n is $\left[\frac{n}{2}\right] + 1$, that is, there are $\left[\frac{n}{2}\right] + 1$ polynomials satisfying the conditions of the problem.

6. First note that every student has a friend. Assume that students $A_1, A_2, \ldots, A_k$ are friends of each other, where k is a positive integer, $2 \le k \le m$. Then there is a student A_{k+1} who is a common friend to all of the students A_i, $1 \le i \le k$. Thus, we can start with a pair of students A_1, A_2 who are friends, and keep adding a student until we obtain $m + 1$ students $A_1, A_2, \ldots, A_{m+1}$ who are friends of each other.

We claim that there are no students other than $A_1, A_2, \ldots, A_{m+1}$ in the camp. For the sake of contradiction, assume there is another student B in the camp. Then B must have a friend. We consider the following situations.

Case-I : If B has atleast two friends among the students $A_1, A_2, \ldots, A_{m+1}$, we assume without loss of generality that A_1 and A_2 are friends of B. Then the m students

$B, A_3, A_4, ..., A_{m+1}$ have two common friends A_1 and A_2, which contradicts the conditions of the problem.

Case-II : If B has no more than one friend among the students $A_1, A_2, ..., A_{m+1}$ in the camp, we assume without loss of generality that $A_2, A_3, ..., A_{m+1}$ are not friends of B. Then the m students $B, A_1, A_3, ..., A_m$ have a common friend C, $C \neq A_i$ for $1 \leq i \leq m+1$. But since $m \geq 3$, student C has at least 2 friends among the students $A_1, A_2, ..., A_{m+1}$. But this impossible by our argument in Case-I.

Overall, we showed that this camp only has the $m+1$ students $A_1, A_2, ..., A_{m+1}$ and they are all friends of each other. Hence the desired number is m.

7. We will find a general formula. Let $A_1A_2 \ldots A_n$ ($n \geq 3$) be a regular n-sided polygon with O as its center. Triangular regions OA_iA_{i+1}, $1 \leq i \leq n$ (and $A_{n+1} = A_1$) are to be colored in one of the k ($k \geq 3$) colors such that adjacent regions are colored in different colors. Let $p_{n,k}$ denote the number of such colorings. We want to find $p_{12,4}$.

There are k ways to color the region OA_1A_2, and then $k-1$ ways to color regions OA_2A_3, OA_3A_4, and so on. We have to be careful about the coloring of the region OA_nA_1. It is possible that it has the same color as that of region OA_1A_2. But then, we simply end up with a legal coloring for $n-1$ regions by viewing region OA_nA_2 as one region. This is a clear bijection between this special kind of illegal colorings of n regions to legal colorings of $n-1$ regions. Hence $p_{n,k} = k(k-1)^{n-1} - p_{n-1,k}$. Note that $p_{3,k} = k(k-1)(k-2)$. If follows that

$$p_{n,k} = k(k-1)^{n-1} - k(k-1)^{n-2} + k(k-1)^{n-3} - \ldots + (-1)^{n-4}k(k-1)^3 + (-1)^{n-3}k(k-1)(k-2)$$

$$= k \cdot \frac{(k-1)^n + (-1)^{n-4}(k-1)^3}{1+(k-1)} + (-1)^{n-3}k(k-1)(k-2)$$

$$= (k-1)^n + (-1)^n(k-1)^3 + (-1)^{n-1}k(k-1)(k-2)$$

$$= (k-1)^n + (-1)^n(k-1)[(k-1)^2 - k(k-2)]$$

$$= (k-1)^n + (-1)^n(k-1).$$

Hence $p_{12,4} = 3^{12} + 3 = 531,444$ ways to color this regular dodecagon.

8. (i) For $n = 1$, $3^2 \mid (2^3 + 1)$ and so the statement is true for $n = 1$.

Suppose $3^{k+1} \mid \left(2^{3^k} + 1\right)$ for some k is true. Now, we have to show that $3^{k+2} \mid \left(2^{3^{k+1}} + 1\right)$.

Consider $3^{k+1} \mid (2^{3^k} + 1) \Rightarrow (3^{k+1})^3 \mid \left(2^{3^k} + 1\right)^3$

i.e., $3^{3k+3} \mid \left(2^{3^{k+1}} + 1 + 3 \cdot 2^{3^k}\left(2^{3^k} + 1\right)\right)$

Since $3k+3 > k+2$, so the above expression implies that

$$3^{k+2} \mid \left(2^{3^{k+1}} + 1 + 3 \cdot 2^{3^k}\left(2^{3^k} + 1\right)\right)$$

But $3^{k+1} \mid \left(2^{3^k} + 1\right)$ by the induction hypothesis and hence $3^{k+2} \mid 3\left(2^{3^k} + 1\right)$.

Hence $3^{k+2} \mid \left(2^{3^{k+1}} + 1\right)$ and we are through.

(ii) Here the induction hypothesis is that for any n, 3^{n+2} does not divide $2^{3^n} + 1$.

For $n = 1$ it is true that 3^3 does not divide $2^3 + 1$.

Suppose that 3^{k+2} does not divide $2^{3^k} + 1$ for some k. Proceeding as in (i) we get that

$$3^{k+3} \mid \left(2^{3^{k+1}} + 1 + 3 \cdot 2^{3^k}\left(2^{3^k} + 1\right)\right)$$

Now if 3^{k+3} divides $2^{3^{k+1}} + 1$, then 3^{k+3} divides $3 \cdot 2^{3^k}(2^{3^k} + 1)$ which in turn implies that 3^{k+2} divides $2^{3^k} + 1$ which is a contradiction to the induction hypothesis. Therefore 3^{k+3} does not divide $2^{3^{k+1}} + 1$. and that finishes the proof.

9. Since the product of k consecutive integers is divisible by $k!$, $A(n)$ is an integer. We compare the highest powers of 2 dividing the numerator and denominator to determine the nature of $A(n)$.

Suppose we express n in the base 2, say,

$n = a_l 2^l + a_{l-1}2^{l-1} + a_{l-2}2^{l-2} + \ldots + a_1 \cdot 2 + a_0$, $a_l = 1$.

The highest power of 2 dividing $n!$ is given by

$$s = \left[\frac{n}{2}\right] + \left[\frac{n}{2^2}\right] + \left[\frac{n}{2^3}\right] + \ldots + \left[\frac{n}{2^l}\right]$$

where $[x]$ denotes the largest integer smaller than x. But, for $1 \leq m \leq l$,

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$$\left[\frac{n}{2^m}\right] = a_l 2^{l-m} + a_{l-1}2^{l-m-1} + \ldots + a_m$$

Thus, $s = \sum_{m=1}^{l}\left[\frac{n}{2m}\right] = \sum_{m=1}^{l}\sum_{k=m}^{l} a_k 2^{k-m}$

$$= \sum_{k=1}^{l} a_k\left(\sum_{m=1}^{l} 2^m - 1\right) = \sum_{k=1}^{l} a_k(2^k - 1)$$

$$= \sum_{k=0}^{l} a_k 2^k - \sum_{k=0}^{l} a_k$$

$= n - \{$ sum of the digits of n in the base 2$\}$

$\because$ For any prime p, the highest power s of p dividing $n!$ is given by

$$s = \frac{n - (\text{sum of the digits of } n \text{ in the base } p)}{p-1}$$

Hence, the highest power of 2 dividing $(n!)^2$ is $2s = 2n - 2$(sum of the digits of n in the base 2). Similarly the highest power of 2 dividing $(2n)!$ is $t = 2n -$ (sum of the digits of $2n$ in the base 2). But the digits of n in base 2 and those of $2n$ in base 2 are the same except for a zero at the end of the representation for $2n$.

Thus $t - 2s = a_l + a_{l-1} + a_{l-2} + \ldots\ldots + a_1 + a_0$

where the a_i are the digits of n in base 2. Note that $a_l = 1$. Hence $t - 2s \geq 1$. But $A(n)$ is even if and only if $t - 2s \geq 1$. Hence it follows that $A(n)$ is even for all n. Moreover $A(n)$ is divisible by 4 if and only if $t - 2s \geq 2$. Since $A(1) = 2$, 4 does not divide $A(1)$. Suppose $n = 2^l$ for some l. Then $a_l = 1$ and $a_i = 0$ for $0 \leq i \leq l - 1$. Hence $t - 2s = 1$ and $A(n)$ is not divisible by 4. On the other hand if n is not a power of 2, then for some l,

$$n = 2^l + a_{l-1}2^{l-1} + a_{l-2}2^{l-2} + \ldots\ldots + a_0$$

where $a_i \neq 0$ for at least one i and hence must be equal to 1. Thus $t - 2s \geq 1 + a_i \geq 2$. It follows that $A(n)$ is divisible by 4 if and only if n is not a power of 2.

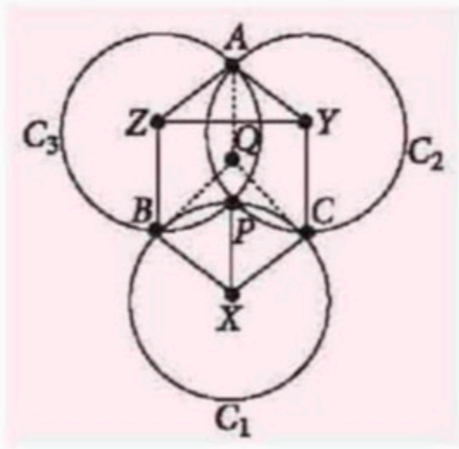
10. We need to know the following facts about three equal circles intersecting in a common point. If three congruent (that is, equal) circles C_1, C_2, C_3 have a common point P and A, B, C are the other three points of intersections, then

(a) the circumcircle of triangle ABC has the same radius as the three circles;

(b) the point P is the orthocentre of triangle ABC.

A brief proof of (a) and (b) follows:

Let X, Y, Z be the centres of the circles C_1, C_2, C_3 respectively. Complete the quadrilaterals $PXBZ$ and $PXCY$, join AP and ZY. Observe that $PXBZ$ and $PXCY$ are rhombuses and so ZB is parallel and equal to YC. Hence so are BC and ZY. Since AP is perpendicular to ZY, AP is perpendicular to BC. Similarly BP and CP are perpendicular to CA and AB respectively. Hence P is the orthocentre of triangle ABC. This proves (b)

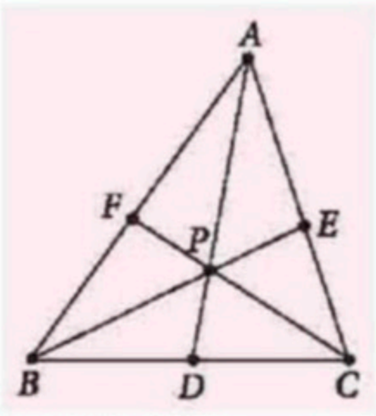


To prove (a), complete the parallelogram $AYCQ$, which is in fact a rhombus. So $AQ = CQ$. It is easily seen that $AZBQ$ is also a rhombus. So $AQ = BQ$. Thus Q is circumcentre of triangle ABC and its radius $(= AQ = CY)$ is the same as that of each of the three circles. Note that we can have a configuration of three equal circles such that P falls outside triangle ABC, but statements (a) and (b) are still true.

Now, let (XYZ) denote the circle through any three non collinear points X, Y, Z. It is given that three equal circles pass through P. Hence by (a) above, the four circles (PAB), (PBC), (PCA) and (ABC) are congruent to one another. Observe that either the three circles (PAB), (PBC), (PCA) coincide [and hence coincide with (ABC)] or the three circles are all distinct passing through the point P. Thus either P is on the circumcircle of ABC or P is the orthocentre of ABC.

11. Applying Ceva's theorem to the Cevians AD, BE, CF which are concurrent at P, we have

$$\frac{AF}{FB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA} = 1$$



Therefore, $\frac{AE}{EC} = \frac{AF}{FB}\cdot\frac{BD}{DC}$

Hence $\frac{AF}{FB} + \frac{AE}{EC} = \frac{AF}{FB}\left(1 + \frac{BD}{DC}\right) = \frac{AF}{FB}\cdot\frac{BC}{DC}$ ...(1)

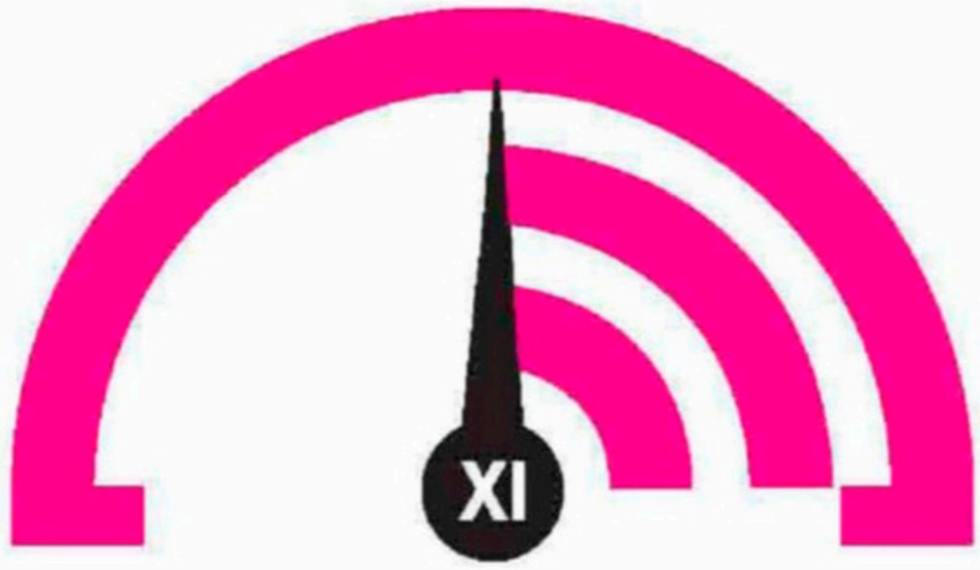
Now applying Menelaus's theorem to triangle ABD, whose sides are cut by the line FPC, we have

$$\frac{AF}{FB}\cdot\frac{BC}{DC}\cdot\frac{DP}{PA} = 1$$

Consequently, $\frac{AF}{FB}\cdot\frac{BC}{DC} = \frac{AP}{PD}$ .....(2)

Comparing (1) and (2), we have the desired relation.

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of concepts of all chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. Let w be the complex number $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Then the number of distinct complex number z satisfying $\begin{vmatrix} z+1 & w & w^2 \\ w & z+w^2 & 1 \\ w^2 & 1 & z+w \end{vmatrix}=0$ is equal to

(a) 1 (b) 0 (c) 2 (d) 3

2. If 1, $\log_9(3^{1-x}+2)$ and $\log_3(4\cdot3^x-1)$ are in A.P., then x is equal to

(a) $\log_4 3$ (b) $\log_3 4$ (c) $1-\log_3 4$ (d) $\log_3 0.25$

3. If $A=\{\theta : 2\cos^2\theta+\sin\theta\le 2\}$ and $B=\left\{\theta:\frac{\pi}{2}\le\theta\le\frac{3\pi}{2}\right\}$, then $A\cap B$ is equal to

(a) $\{\theta : \pi/2\le\theta\le 5\pi/6\}$
(b) $\{\theta : \pi\le\theta\le 3\pi/2\}$
(c) $\{\theta : \pi/2\le\theta\le 5\pi/6\}\cup\{\theta : \pi\le\theta\le 3\pi/2\}$
(d) none of these

4. The value of the expression ${}^{47}C_4+\sum_{j=1}^{5}{}^{52-j}C_3$, is

(a) ${}^{52}C_4$ (b) ${}^{52}C_3$ (c) ${}^{51}C_4$ (d) ${}^{53}C_4$

5. If x, y, z are non-negative real numbers satisfying $x+y+z=1$, then the minimum value of $\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)\left(\frac{1}{z}+1\right)$, is

(a) 8 (b) 16 (c) 32 (d) 64

6. If $x\sin\theta=y\sin\left(\theta+\frac{2\pi}{3}\right)=z\sin\left(\theta+\frac{4\pi}{3}\right)$, then

(a) $x+y+z=0$ (b) $xy+yz+zx=0$
(c) $xyz+x+y+z=1$ (d) none of these

One or More Than One Option(s) Correct Type

7. The real values of 'a' for which the equation $x^4-2x^2a-x^2+6x+a^2-3a=0$ has all its roots real

(a) $a>4$ (b) $a\ge 2$ (c) $a\ge 3/4$ (d) $a\ge -1$

8. If $f(x)=e^{[\cot x]}$, where $[\cdot]$ denotes the greatest integer function, then

(a) $\lim\limits_{x\to\pi/2^+} f(x)=1$ (b) $\lim\limits_{x\to\pi/2^+} f(x)=\frac{1}{e}$
(c) $\lim\limits_{x\to\pi/2^-} f(x)=1$ (d) $\lim\limits_{x\to\pi/2^-} f(x)=\frac{1}{e}$

9. If p, q and r are simple proposition, then $(\sim p\vee q)\Rightarrow r$ is true, when p, q and r, respectively are

(a) T, F, T (b) T, T, T
(c) T, F, F (d) F, F, F

10. Out of $3n$ consecutive integers, three are selected at random. The probability that their sum is divisible by 3, is

(a) $\frac{n(3n^2-3n+2)}{2}$ (b) $\frac{3n^2-3n+2}{2(3n-1)(3n-2)}$
(c) $\frac{3n^2-3n+2}{(3n-1)(3n-2)}$ (d) $\frac{n(3n-1)(3n-2)}{3(n-1)}$

11. The equation of a circle C_1 is $x^2+y^2=4$. The locus of the intersection of orthogonal tangents to the circle is the curve C_2 and the locus of the intersection of perpendicular tangents to the curve C_2 is the curve C_3. Then,

(a) C_3 is a circle
(b) the area enclosed by the curve C_3 is 8π
(c) C_2 and C_3 are circles with the same centre
(d) none of these

12. Let $\theta, \varphi \in [0, 2\pi]$ such that

$$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1,$$

$\tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$.

Then φ cannot satisfy

(a) $0 < \varphi < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$

(c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (d) $\frac{3\pi}{2} < \varphi < 2\pi$

13. Range of $f(x) = \frac{1}{1-2\cos x}$ is

(a) $\left[\frac{1}{3}, 1\right]$ (b) $\left[-1, \frac{1}{3}\right]$

(c) $(-\infty, -1) \cup \left[\frac{1}{3}, \infty\right)$ (d) $\left[-\frac{1}{3}, 1\right]$

Comprehension Type

Let us consider the series $x_1, x_2, ..., x_n$ whose mean is $\bar{x}$ and variance is σ^2.

On the basis of above information, answer the following questions :

14. If x_i is replaced by x_i', then new mean is

(a) $\bar{x} - x_i + x_i'$ (b) $\frac{(n-1)\bar{x} + x_i'}{n}$

(c) $\frac{n\bar{x} - x_i + x_i'}{n}$ (d) none of these

15. If 5 is added in each observation, then the new variance is

(a) σ^2 (b) $\sigma^2 + 5$

(c) $\sigma^2 - 5$ (d) none of these

Matrix Match Type

16. Match the following :

Column-I		Column-II	
P.	The locus of mid-points of chords of an ellipse which are drawn through an end of minor axis, is	1.	hyperbola
Q.	The locus of an end of latus rectum of all ellipses having a given major axis is	2.	circle
R.	The locus of the foot of perpendicular from a focus of the ellipse on any tangent is	3.	parabola
S.	A variable line is drawn through a fixed point cuts axes at A and B. The locus of the mid point of AB is	4.	ellipse

	P	Q	R	S
(a)	3	4	1	2
(b)	4	3	2	1
(c)	2	4	3	1
(d)	4	2	1	3

Numerical Value Type

17. In a ΔABC, $a = 5$, $b = 4$ and $\tan\frac{C}{2} = \sqrt{\frac{7}{9}}$, then $c =$ _____.

18. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}\frac{3}{4}$ with the x-axis. It intersects the parabola $y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. If $|x_1 - x_2| = k/9$, then k equals _____.

19. Let $A(2, 6)$ and $B(8, 8)$ are two points. Starting from A, line segments of unit length are drawn either right wards or upwards only, untill B is reached, then the number of ways in which A and B connect is 7×2^k. The value of k equals _____.

20. If $f(x) = \frac{x-4}{2\sqrt{x}}$, then $f'(1) =$ _____.

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SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

MONTHLY TEST DRIVE

This specially designed column enables students to self analyse their extent of understanding of concepts of all chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $f: \{1, 2, 3, 4\} \to \{1, 4, 9, 16\}$ and $g: \{1, 4, 9, 16\} \to \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$ are two bijective functions such that $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$, $g(x_1) > g(x_2)$, then $f^{-1}\left(g^{-1}\left(\frac{1}{2}\right)\right)$ is equal to

(a) 1 (b) 4 (c) 16 (d) 2

2. The value of c in Lagrange's theorem for the function $f(x) = \log \sin x$ in the interval $[\pi/6, 5\pi/6]$ is

(a) $\pi/4$ (b) $\pi/2$
(c) $2\pi/3$ (d) none of these

3. The area bounded by the curve $[x] + [y] = 4$ in first quadrant is (where $[\cdot]$ denotes the greatest integer function)

(a) 13 sq. units (b) 14 sq. units
(c) 15 sq. units (d) 16 sq. units

4. A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing card is black, is

(a) $\frac{2}{3}$ (b) $\frac{15}{26}$ (c) $\frac{16}{39}$ (d) $\frac{37}{52}$

5. If $f(x) = (x-p)(x-q)(x-r)$, where $p < q < r$, are real numbers, then the application of Rolle's theorem on f leads to

(a) $(p + q + r)(pq + qr + rp) = 3$
(b) $(p + q + r)^2 = 3(pq + qr + rp)$
(c) $(p + q + r)^2 > 3(pq + qr + rp)$
(d) $(p + q + r)^2 < 3(pq + qr + rp)$

6. If $\vec{u}$ and $\vec{v}$ are unit vectors and θ is the acute angle between them, then $2\vec{u} \times 3\vec{v}$ is a unit vector for

(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

One or More Than One Option(s) Correct Type

7. $\int_{-1/2}^{1/2} \sqrt{\left\{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right\}} dx$ is equal to

(a) $4\ln\left(\frac{4}{3}\right)$ (b) $4\ln\left(\frac{3}{4}\right)$
(c) $-\ln\left(\frac{81}{256}\right)$ (d) $\ln\left(\frac{256}{81}\right)$

8. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

(a) $f(x)$ is not continuous at $x = 0$
(b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$
(d) $f(x)$ is differentiable at $x = 0$

9. Which of the following statements is/are true about square matrix A of order n?

(a) $(-A)^{-1}$ is equal to $-A^{-1}$ when n is odd only.
(b) If $A^n = O$, then $I + A + A^2 + \ldots + A^{n-1} = (I - A)^{-1}$.
(c) If A is skew-symmetric matrix of odd order, then its inverse does not exist.
(d) $(A^T)^{-1} = (A^{-1})^T$ holds if $|A| \neq 0$.

10. If $K=\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)$ and $\frac{\pi}{4}<A<\frac{\pi}{2}$ then $[K]=$

(where [.] denotes greatest integer function)

(a) 1 (b) 0 (c) 2 (d) 3

11. $f_n(x)=e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is

(a) $f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$ (b) $f_n(x)f_{n-1}(x)$

(c) $f_n(x)\cdot f_{n-1}(x)...f_2(x)\cdot f_1(x)$

(d) none of these

12. If $x\frac{dy}{dx}+y=x\cdot\frac{f(xy)}{f'(xy)}$, then $f(xy)$ is equal to

(a) $ke^{\frac{x^2}{2}}$ (b) $ke^{\frac{y^2}{2}}$

(c) $ke^{c+\frac{x^2}{2}}$ (d) $ke^{\frac{xy}{2}}$

13. If $OABC$ is a tetrahedron such that $OA^2 + BC^2 = OB^2 + CA^2 = OC^2 + AB^2$, then

(a) $OA \perp BC$ (b) $OB \perp CA$

(c) $OC \perp AB$ (d) $AB \perp BC$

Comprehension Type

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day, and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively.

14. The chance she will be successful is

(a) 0.28 (b) 0.38

(c) 0.48 (d) 0.58

15. Given that she is successful, the chance that she studies for 4 hours is

(a) 6/12 (b) 7/12

(c) 8/12 (d) 9/12

Matrix Match Type

16. Consider the linear equations $ax + by + cz = 0$, $bx + cy + az = 0$ and $cx + ay + bz = 0$. Match the conditions in column-I with statements in column-II.

Column-I		Column-II	
P.	$a+b+c\neq 0$ and $a^2+b^2+c^2=ab+bc+ca$	1.	The equations represent planes meeting only at a single point.
Q.	$a+b+c=0$ and $a^2+b^2+c^2\neq ab+bc+ca$	2.	The equations represent the line $x=y=z$.
R.	$a+b+c\neq 0$ and $a^2+b^2+c^2\neq ab+bc+ca$	3.	The equations represent identical planes.
S.	$a+b+c=0$ and $a^2+b^2+c^2=ab+bc+ca$	4.	The equations represent the whole of the three-dimensional space.

	P	Q	R	S
(a)	2	1	3	4
(b)	1	3	2	3
(c)	3	2	1	4
(d)	2	4	1	3

Numerical Value Type

17. Let $F(x)=\int_x^{x+3} 2t(5-t)dt$. If M is the maximum value of $F(x)$, then the sum of the digits of M is _____.

18. If $\vec{a}+2\vec{b}+3\vec{c}=0$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=k\vec{a}\times\vec{b}$, where k is equal to _____.

19. Given $2x-y+2z=2$, $x-2y-z=-4$, $x+y+\lambda z=4$, then the value of λ such that the given system of equations has no solution is _____.

20. Absolute value of sum of all integers in the domain of $f(x)=\cot^{-1}\sqrt{(x+3)x}+\cos^{-1}\sqrt{x^2+3x+1}$ is _____.

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SELF CHECK

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No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

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< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Let N denotes the greatest number of points in which m straight lines and n circles intersect, then prove that $N - {}^mC_2 - {}^nP_2$ is an even integer.

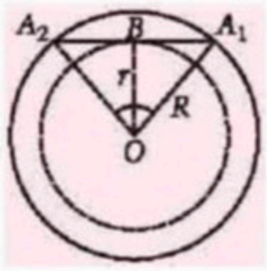
(Pratik, W.B.)

Ans. m straight lines can intersect in at most ${}^mC_2$ points and n circles can intersect in at most $2 \times {}^nC_2 = n(n-1) = {}^nP_2$ points.
n straight lines and m circles can intersect in at most $2 \times m \times n = 2mn$ points
Hence, $N = {}^mC_2 + {}^nP_2 + 2mn$
$\Rightarrow (N - {}^mC_2 - {}^nP_2) = 2mn$, which is an even integer.

2. If a regular polygon of n sides has the circumradius R and inradius r, then prove that each side of the polygon is equal to $2(R+r)\tan\frac{\pi}{2n}$.

(Suresh, Gujarat)

Ans. Let A_1A_2 be a side and O be the centre. Let $OB \perp A_1A_2$.



Clearly, $OA_1 = R$ and $OB = r$.

Also, $\angle A_1OA_2 = \frac{2\pi}{n}$ and $\angle A_1OB = \frac{\pi}{n}$.

In ΔA_1OB, $\frac{r}{R} = \cos\angle A_1OB = \cos\frac{\pi}{n} \Rightarrow R = \frac{r}{\cos\frac{\pi}{n}}$ ...(i)

and $\frac{A_1B}{OB} = \tan\angle A_1OB = \tan\frac{\pi}{n}$, $\Rightarrow A_1B = r\tan\frac{\pi}{n}$

$\therefore \quad A_1A_2 = 2A_1B = 2r\tan\frac{\pi}{n}$ ...(ii)

Now, $2(R+r)\tan\frac{\pi}{2n} = 2\left(\frac{r}{\cos\frac{\pi}{n}} + r\right)\tan\frac{\pi}{2n}$ (From (i))

$= 2r\left(\frac{1+\cos\frac{\pi}{n}}{\cos\frac{\pi}{n}}\right).\frac{1-\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} \quad \left\{\because \tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}\right\}$

$= 2r\frac{1-\cos^2\frac{\pi}{n}}{\cos\frac{\pi}{n}\cdot\sin\frac{\pi}{n}} = 2r\cdot\frac{\sin^2\frac{\pi}{n}}{\cos\frac{\pi}{n}\cdot\sin\frac{\pi}{n}}$

$= 2r\tan\frac{\pi}{n} = A_1A_2$ (From (ii))

3. $\hat{n}$ is a unit vector perpendicular to the plane containing the points whose position vectors are $\vec{a}$, $\vec{b}$ and $\vec{r}$, satisfying $|[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \hat{n}]| = \lambda|\vec{r}\times(\vec{b}-\vec{a})-(\vec{a}\times\vec{b})|$, then find the value of λ.

(Ram Krishan, Patna)

Ans. $(\vec{r}-\vec{a})\times(\vec{b}-\vec{a})$ is parallel to $\hat{n}$

$\Rightarrow \quad (\vec{r}-\vec{a})\times(\vec{b}-\vec{a}) = t\hat{n}$ ...(i)

$\therefore \quad |(\vec{r}-\vec{a})\times(\vec{b}-\vec{a})\cdot\hat{n}| = |t|$

$\Rightarrow \quad |[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \hat{n}]| = |t|$ ...(ii)

From (i), $\vec{r}\times(\vec{b}-\vec{a}) - \vec{a}\times\vec{b} = t\hat{n}$

$\Rightarrow \quad |\vec{r}\times(\vec{b}-\vec{a})-(\vec{a}\times\vec{b})| = |t|$ ...(iii)

From (ii) and (iii), we get

$|[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \hat{n}]| = |\vec{r}\times(\vec{b}-\vec{a})-(\vec{a}\times\vec{b})|$

$\therefore \quad \lambda = 1$

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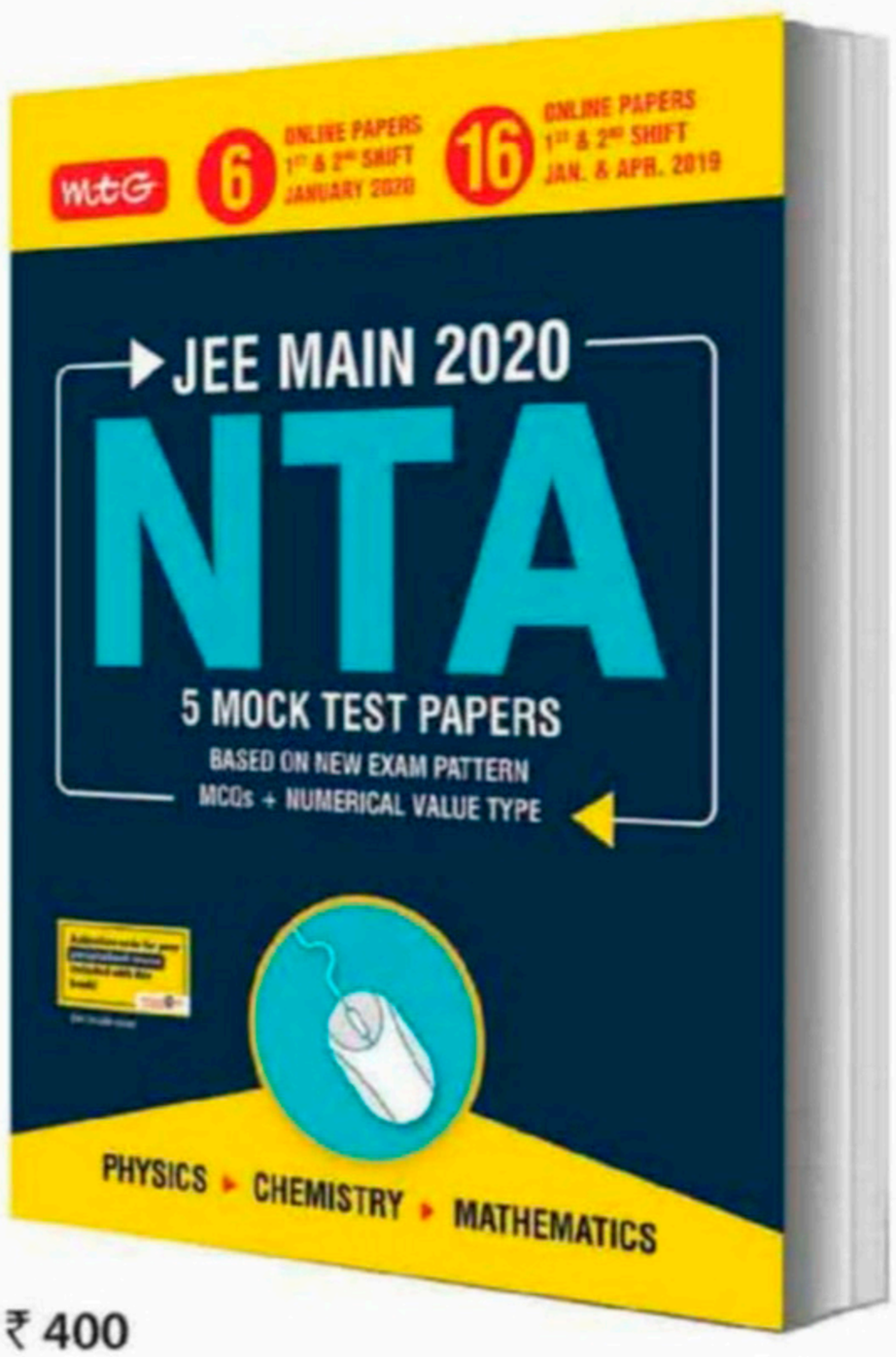

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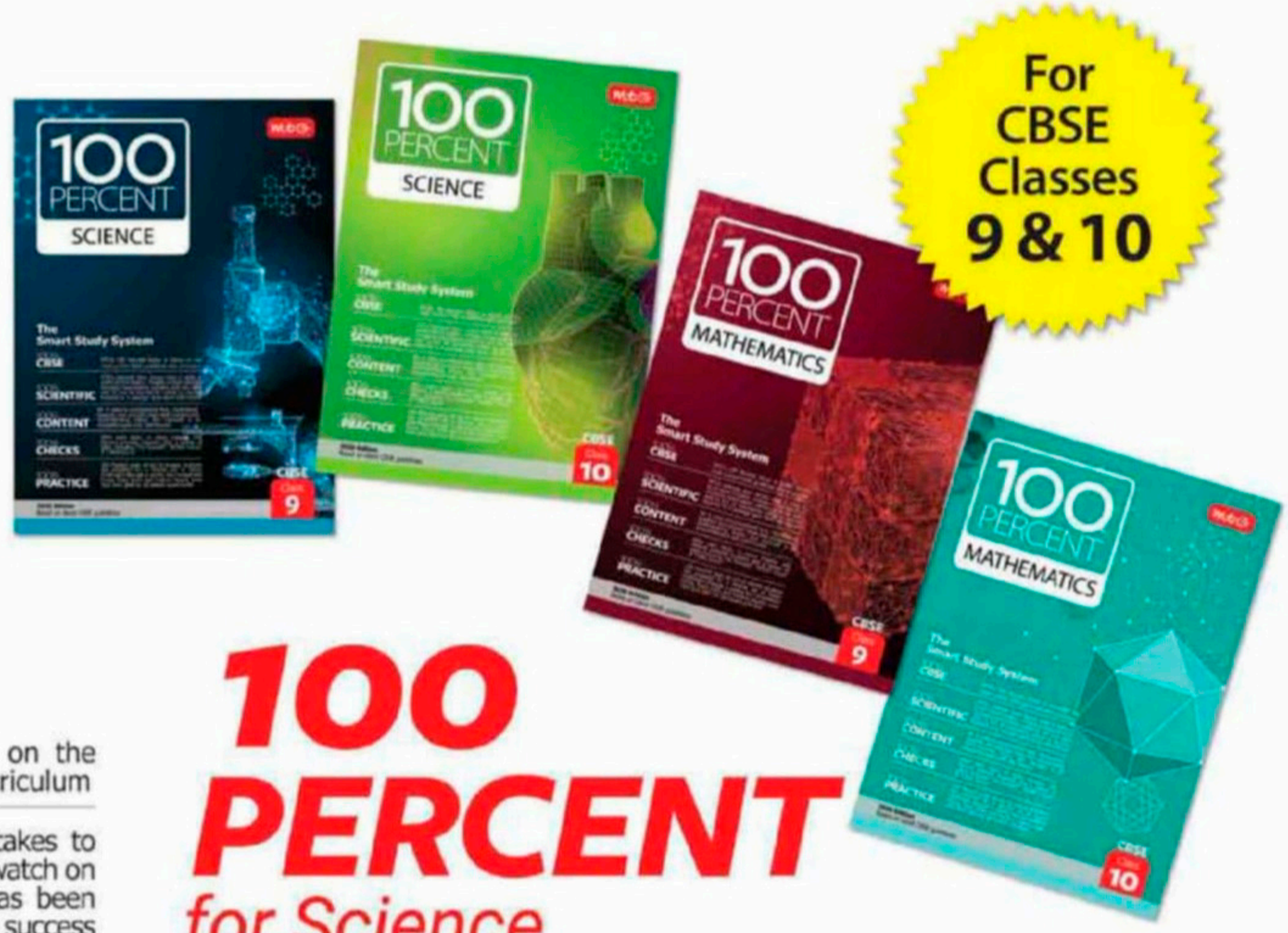

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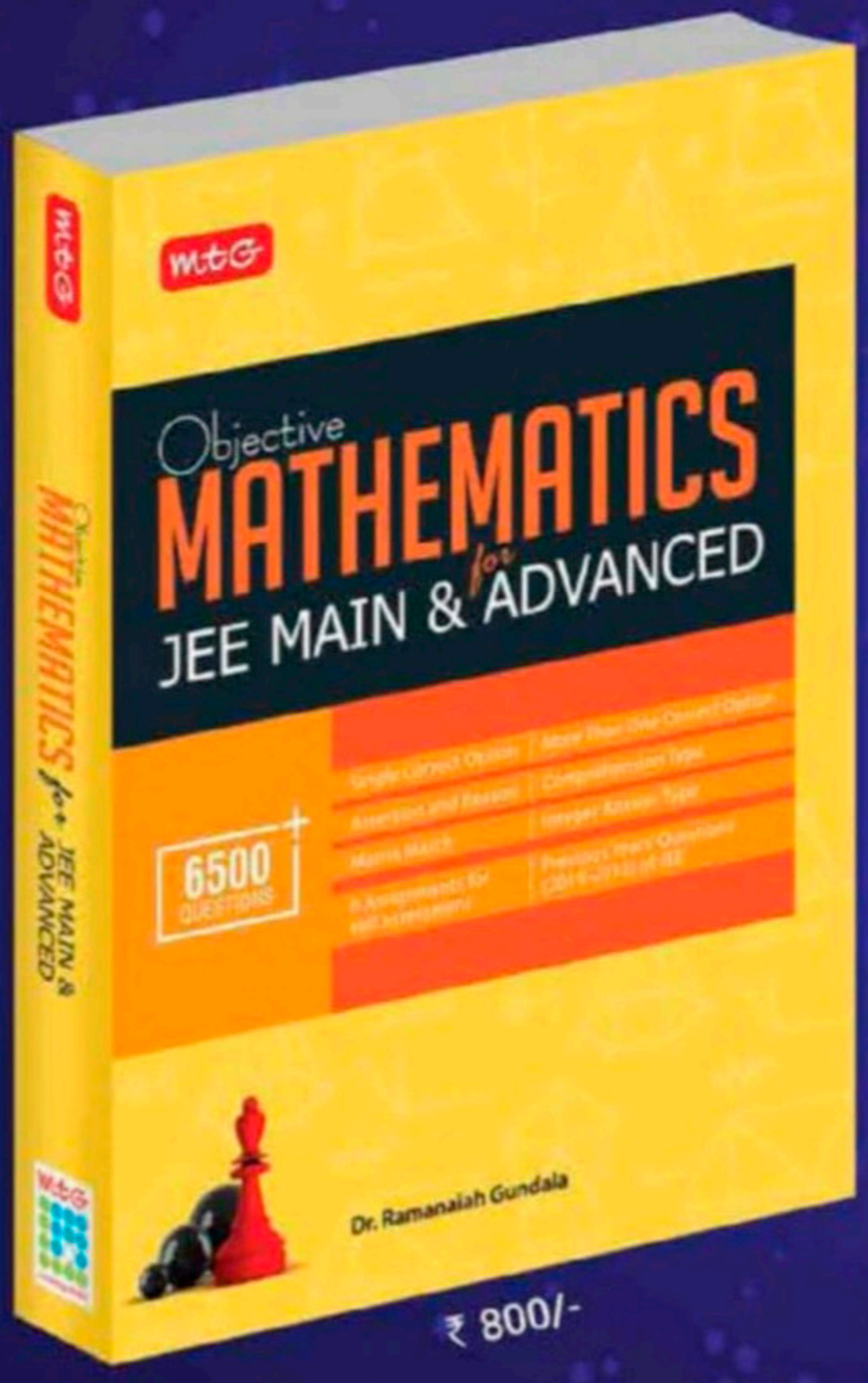

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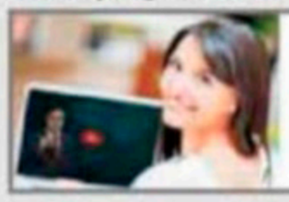
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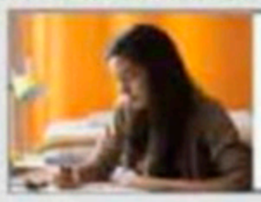
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